

Question #2

- a.) $\log_5 25 = 2$
- b.) $\log_2 \frac{1}{8} = -3$
- c.) $\log_7 \sqrt{7} = \frac{1}{2}$
- d.) $\log_5 5 = 1$
- e.) $\log_0 1 = 0$
- f.) $\log_4 4^6 = 6$
- g.) $8^{\log_8 19} = 19$
- h.) $\log_2 64 = 6$
- i.) $\log_{25} 7 = \frac{1}{2}$

Question #3

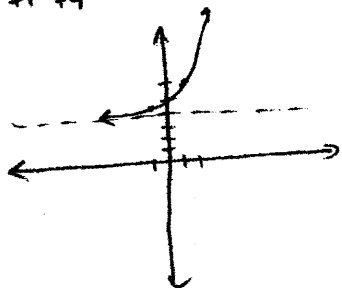
- a.) $\log 52 \approx 1.716$
- b.) $\ln 28 \approx 3.332$
- c.) $\log_5 12 = \frac{\ln 12}{\ln 5} \approx 1.544$
- d.) $\log_7 31 = \frac{\ln 31}{\ln 7} \approx 1.765$

Question #1

- a.) $f(x) = 2^{x-1} + 4$
 shift 1 unit right
 shift 4 units up

2^x	$2^{x-1} + 4$
$(-1, \frac{1}{2})$	$(0, 4\frac{1}{2})$
$(0, 1)$	$(1, 5)$
$(1, 2)$	$(2, 6)$

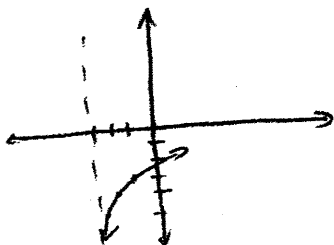
↑ ↑
+1 +4



- b.) $\log_2 (x+3) - 4$
 shift 3 units left
 shift 4 units down

2^x	$\log_2 x$	$\log_2 (x+3) - 4$
$(-1, \frac{1}{2})$	$(\frac{1}{2}, -1)$	$(-2\frac{1}{2}, -5)$
$(0, 1)$	$(1, 0)$	$(-2, -4)$
$(1, 2)$	$(2, 1)$	$(-1, -3)$

↑ ↑
-3 -4



Question #4

- a.) $P = \$50, r = 6\%, n = 12, t = 3$

$$A = P(1 + \frac{r}{n})^{nt}$$

$$A = 50(1 + \frac{0.06}{12})^{12(3)}$$

$$A = \$59.83$$

- b.) $P = \$700, r = 6\%, n = 365, t = 2$

$$A = P(1 + \frac{r}{n})^{nt}$$

$$A = 700(1 + \frac{0.06}{365})^{365(2)}$$

$$A = \$789.24$$

- c.) $P = \$100, r = 12\%, t = 3\frac{3}{4}$

$$A = Pe^{rt}$$

$$A = 100e^{0.12(3.75)}$$

$$A = \$156.83$$

Question #5

- a.) $2^{2x+1} = 4$
 $2^{2x+1} = 2^2$
 $2x+1 = 2$
 $2x = 1$
 $x = \frac{1}{2}$

- b.) $3^{x^3} = 9^x$
 $3^{x^3} = (3^2)^x$
 $3^{x^3} = 3^{2x}$
 $x^3 = 2x$

$$x^3 - 2x = 0$$

$$x(x^2 - 2) = 0$$

$$x = 0 \quad x^2 - 2 = 0$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

$$x = 0, \pm \sqrt{2}$$

- c.) $5^{2x-1} = 125$

$$5^{2x-1} = 5^3$$

$$2x-1 = 3$$

$$2x = 4$$

$$x = 2$$

- d.) $4^x = \frac{1}{\sqrt{2}}$

$$(2^2)^x = 2^{-1/2}$$

$$2^{2x} = 2^{-1/2}$$

$$2x = -1/2$$

$$x = -1/4$$

- e.) $3^x = 14$

$$\ln 3^x = \ln 14$$

$$x \ln 3 = \ln 14$$

$$x = \frac{\ln 14}{\ln 3}$$

- f.) $3^{1-2x} = 4^x$

$$\ln 3^{1-2x} = \ln 4^x$$

$$(1-2x) \ln 3 = x \ln 4$$

$$\ln 3 - 2x \ln 3 = x \ln 4$$

$$\ln 3 = x \ln 4 + 2x \ln 3$$

$$\ln 3 = x(\ln 4 + 2 \ln 3)$$

$$x = \frac{\ln 3}{\ln 4 + 2 \ln 3}$$

Question #5 continued

g.) $e^{5x} = 7$
 $\ln e^{5x} = \ln 7$
 $5x = \ln 7$
 $x = \frac{\ln 7}{5}$

h.) $\frac{9e^x}{9} = \frac{99}{9}$
 $e^x = 11$
 $\ln e^x = \ln 11$
 $x = \ln 11$

i.) $e^{0.08x} = 4$
 $\ln e^{0.08x} = \ln 4$
 $0.08x = \ln 4$
 $x = \frac{\ln 4}{0.08}$

j.) $\log_2(x^2) - \log_2(x-2) = 3$
 $\log_2 \frac{x^2}{x-2} = 3$
 $2^3 = \frac{x^2}{x-2}$
 $8(x-2) = x^2$
 $8x - 16 = x^2$
 $0 = x^2 - 8x + 16$
 $0 = (x-4)(x-4)$
 $x-4 = 0$
 $x = 4$
 check in each log ✓
 $x = 4$

k.) $\log_2(x-3) + \log_2(x+4) = 3$
 $\log_2(x-3)(x+4) = 3$
 $2^3 = (x-3)(x+4)$
 $8 = x^2 + x - 12$
 $0 = x^2 + x - 20$
 $0 = (x-4)(x+5)$
 $x = 4$ or $x = -5$
 does not check

l.) $x = \log_5 625$
 $5^x = 625$
 $5^x = 5^4$
 $x = 4$

m.) $\log_8 x = -2$
 $8^{-2} = x$
 $x = \frac{1}{64}$

n.) $\log_x 4 = \frac{1}{3}$
 $x^{1/3} = 4$
 $(x^{1/3})^3 = (4)^3$
 $x = 64$
 checks

o.) $\log_4(x^2-9) - \log_4(x+3) = 3$
 $\log_4 \frac{x^2-9}{x+3} = 3$
 $4^3 = \frac{(x+3)(x-3)}{x+3}$
 $64 = x-3$
 $x = 67$
 checks

p.) $\log(3x-5) - \log 5x = 2$
 $\log \frac{3x-5}{5x} = 2$
 $10^2 = \frac{3x-5}{5x}$
 $100 = \frac{3x-5}{5x}$
 $500x = 3x-5$
 $497x = -5$
 $x = -\frac{5}{497}$
 Does not check
 \emptyset

Question #6

a.) $\log_3 \frac{x^4 y^3}{z^2 w^3}$
 $\log_3(x^4 y^3) - \log_3(z^2 w^3)$
 $(\log_3 x^4 + \log_3 y^3) - (\log_3 z^2 + \log_3 w^3)$
 $4\log_3 x + 3\log_3 y - 2\log_3 z - 3\log_3 w$

b.) $\log \sqrt{\frac{x^2}{y^3}}$
 $\log \left(\frac{x^2}{y^3}\right)^{1/2}$
 $\log \frac{x}{y^{3/2}}$
 $\log x - \log y^{3/2}$
 $\log x - \frac{3}{2} \log y$

Question #7

a.) $2\log_3 x + 4\log_3 y - 6\log_3 z$
 $\log_3 x^2 + \log_3 y^4 - \log_3 z^6$
 $\log_3(x^2 y^4) - \log_3 z^6$
 $\log_3 \left(\frac{x^2 y^4}{z^6}\right)$

b.) $6\log_4 x - 2\log_4 y + 3\log_4 z$
 $\log_4 x^6 + \log_4 z^3 - \log_4 y^2$
 $\log_4(x^6 z^3) - \log_4 y^2$
 $\log_4 \left(\frac{x^6 z^3}{y^2}\right)$

Question #8

$N(t) = 1000e^{0.01t}$

$N = 1500$	$\frac{1500}{1000} = \frac{1000e^{0.01t}}{1000}$	$1.5 = e^{0.01t}$	$\ln 1.5 = \ln e^{0.01t}$	$\ln 1.5 = 0.01t$	$t = \frac{\ln 1.5}{0.01} = 40.5 \text{ hrs}$
$N = 2000$	$\frac{2000}{1000} = \frac{1000e^{0.01t}}{1000}$	$2 = e^{0.01t}$	$\ln 2 = \ln e^{0.01t}$	$\ln 2 = 0.01t$	$t = \frac{\ln 2}{0.01} = 69.3$

Question #9

$$A(t) = A_0 e^{-0.087t}$$

Find half-life

$$\frac{1}{2} A_0 = \frac{A_0 e^{-0.087t}}{A_0}$$

$$\frac{1}{2} = e^{-0.087t}$$

$$\ln \frac{1}{2} = \ln e^{-0.087t}$$

$$\ln \frac{1}{2} = -0.087t$$

$$t = \frac{\ln \frac{1}{2}}{-0.087} = \boxed{7.97 \text{ days}}$$

Find time for 100g to decay to 10g.

$$\frac{10}{100} = \frac{100 e^{-0.087t}}{100}$$

$$0.1 = e^{-0.087t}$$

$$\ln 0.1 = \ln e^{-0.087t}$$

$$\ln 0.1 = -0.087t$$

$$t = \frac{\ln 0.1}{-0.087} = \boxed{26.47 \text{ days}}$$

Question #10

$$A = A_0 e^{kt}$$

Find k given time to double is 18 months (1.5 yrs)

$$\frac{20000}{10000} = \frac{10000 e^{1.5t}}{10000}$$

$$2 = e^{1.5t}$$

$$\ln 2 = \ln e^{1.5t}$$

$$\ln 2 = 1.5t$$

$$t = \frac{\ln 2}{1.5} = 0.462$$

Find population in 2 years

$$A = 10000 e^{0.462t}$$

$$A = 10000 e^{0.462(2)}$$

$$A = \boxed{25193 \text{ people}}$$

Question #11

$$A = A_0 e^{kt}$$

Find k using half-life.

$$\frac{1}{2} A_0 = \frac{A_0 e^{k(1690)}}{A_0}$$

$$\frac{1}{2} = e^{1690k}$$

$$\ln \frac{1}{2} = \ln e^{1690k}$$

$$\ln \frac{1}{2} = 1690k$$

$$k = \frac{\ln \frac{1}{2}}{1690} = -0.0004101$$

Find amount present after 50 yrs.

$$A = 10 e^{-0.0004101(50)}$$

$$A = \boxed{9.8 \text{ g}}$$

Question #12

$$A = A_0 e^{kt}$$

Find k using half-life

$$\frac{1}{2} A_0 = \frac{A_0 e^{k5600}}{A_0}$$

$$\frac{1}{2} = e^{5600k}$$

$$\ln \frac{1}{2} = \ln e^{5600k}$$

$$\ln \frac{1}{2} = 5600k$$

$$k = \frac{\ln \frac{1}{2}}{5600} = -0.000124$$

Find time using 30% of original is present

$$0.30 A_0 = \frac{A_0 e^{-0.000124t}}{A_0}$$

$$0.30 = e^{-0.000124t}$$

$$\ln 0.3 = \ln e^{-0.000124t}$$

$$\ln 0.3 = -0.000124t$$

$$t = \frac{\ln 0.3}{-0.000124} = \boxed{9709 \text{ yrs}}$$

Question #13

a)
$$\begin{aligned} x - 2y &= 16 \\ y + 3 &= 3x \end{aligned}$$

substitution:

$$\begin{aligned} x &= 16 + 2y \\ y + 3 &= 3(16 + 2y) \\ y + 3 &= 48 + 6y \\ 3 &= 48 + 5y \\ -45 &= 5y \\ y &= -9 \end{aligned}$$

$$\begin{aligned} x - 2(-9) &= 16 \\ x + 18 &= 16 \\ x &= -2 \end{aligned} \quad \boxed{(-2, -9)}$$

b)
$$\begin{aligned} x - 3y &= -6 \\ 3x - 9y &= 9 \end{aligned}$$

elimination

$$\begin{aligned} -3(x - 3y) &= -6(-6) \\ 3x - 9y &= 9 \end{aligned}$$

$$\begin{aligned} -3x + 9y &= 18 \\ 3x - 9y &= 9 \\ \hline 0 &= 27 \end{aligned} \quad \boxed{\emptyset}$$

c)
$$\begin{aligned} 2x + 6y &= 8 \\ 3x + 9y &= 12 \end{aligned}$$

elimination:

$$\begin{aligned} 3(2x + 6y) &= 3(8) \\ -2(3x + 9y) &= -2(12) \end{aligned}$$

$$\begin{aligned} 6x + 18y &= 24 \\ -6x - 18y &= -24 \\ \hline 0 &= 0 \end{aligned}$$

$$\boxed{\{(x, y) \mid 2x + 6y = 8\}}$$

Question #3 continued

d.) $3x - 5y = 11$
 $2x - 6y = 2$

eliminate x

$2(3x - 5y = 11)$
 $-3(2x - 6y = 2)$

$6x - 10y = 22$
 $-6x + 18y = -6$

 $8y = 16$
 $y = 2$

$2x - 6(2) = 2$
 $2x - 12 = 2$
 $2x = 14$
 $x = 7$

(7, 2)

e.) $x + y - z = 6$
 $3x - 2y + z = -5$
 $x + 3y - 2z = 14$

① & ② eliminate z
 $x + y - z = 6$
 $3x - 2y + z = -5$

 $4x - y = 1$

② & ③ eliminate z
 $2(3x - 2y + z = -5)$
 $x + 3y - 2z = 14$

 $6x - 4y + 2z = -10$
 $x + 3y - 2z = 14$

④ $7x - y = 4$

④ & ⑤
 $-1(4x - y = 1)$
 $7x - y = 4$

$-4x + y = -1$
 $7x - y = 4$

 $3x = 3$
 $x = 1$

$4(1) - y = 1$
 $4 - y = 1$
 $-y = -3$
 $y = 3$

$3(1) - 2(3) + z = -5$
 $3 - 6 + z = -5$
 $-3 + z = -5$
 $z = -2$

(1, 3, -2)

f.) $4x - y + 2z = 11$ ①
 $x + 2y - z = -1$ ②
 $2x + 2y - 3z = -1$ ③

① & ② eliminate y
 $2(4x - y + 2z = 11)$
 $x + 2y - z = -1$

$8x - 2y + 4z = 22$
 $x + 2y - z = -1$

 $9x + 3z = 21$

① & ③ eliminate y
 $2(4x - y + 2z = 11)$
 $2x + 2y - 3z = -1$

$8x - 2y + 4z = 22$
 $2x + 2y - 3z = -1$

 $10x + z = 21$

④ & ⑤
 $9x + 3z = 21$
 $-3(10x + z = 21)$

$9x + 3z = 21$
 $-30x - 3z = -63$

 $-21x = -42$
 $x = 2$

$10(2) + z = 21$
 $20 + z = 21$
 $z = 1$

$4(2) - y + 2(1) = 11$
 $8 - y + 2 = 11$
 $-y + 10 = 11$
 $-y = 1$
 $y = -1$

(2, -1, 1)

Question #4

a.) $2x - 4y = -2$
 $3x + 2y = 3$

$1 \rightarrow \begin{bmatrix} 2 & -4 & -2 \\ 3 & 2 & 3 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & -2 & -1 \\ 3 & 2 & 3 \end{bmatrix}$

$\xrightarrow{-3R_1 + R_2} \begin{bmatrix} 1 & -2 & -1 \\ 0 & 8 & 6 \end{bmatrix} \xrightarrow{\frac{1}{8}R_2} \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & \frac{3}{4} \end{bmatrix}$

$x - 2y = -1$
 $y = \frac{3}{4}$

$x - 2(\frac{3}{4}) = -1$
 $x - \frac{3}{2} = -1$
 $x = \frac{1}{2}$

($\frac{1}{2}, \frac{3}{4}$)

b.) $2x + y - 3z = 0$
 $-2x + 2y + z = -7$
 $3x - 4y - 3z = 7$

$\begin{bmatrix} 2 & 1 & -3 & 0 \\ -2 & 2 & 1 & -7 \\ 3 & -4 & -3 & 7 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{3}{2} & 0 \\ -2 & 2 & 1 & -7 \\ 3 & -4 & -3 & 7 \end{bmatrix}$

$\xrightarrow{2R_1 + R_2} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{3}{2} & 0 \\ 0 & 3 & -2 & -7 \\ 3 & -4 & -3 & 7 \end{bmatrix}$
 $2R_1 \quad 21 - 3 \quad 0$
 $\quad \quad 22 \quad 1 - 7$
 $\quad \quad 0 \quad 3 - 2 - 7$

$\xrightarrow{-3R_1 + R_3} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{3}{2} & 0 \\ 0 & 3 & -2 & -7 \\ 0 & -\frac{11}{2} & \frac{3}{2} & 7 \end{bmatrix} \xrightarrow{\frac{1}{3}R_2} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{3}{2} & 0 \\ 0 & 1 & -\frac{2}{3} & -\frac{7}{3} \\ 0 & -\frac{11}{2} & \frac{3}{2} & 7 \end{bmatrix}$

$\xrightarrow{\frac{11}{2}R_2 + R_3} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{3}{2} & 0 \\ 0 & 1 & -\frac{2}{3} & -\frac{7}{3} \\ 0 & 0 & -\frac{13}{6} & \frac{35}{6} \end{bmatrix} \xrightarrow{\frac{6}{-13}R_3} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{3}{2} & 0 \\ 0 & 1 & -\frac{2}{3} & -\frac{7}{3} \\ 0 & 0 & 1 & -\frac{35}{13} \end{bmatrix}$

$x + \frac{1}{2}y - \frac{3}{2}z = 0$
 $y - \frac{2}{3}z = -\frac{7}{3}$
 $z = \frac{35}{13}$

$y - \frac{2}{3}(\frac{35}{13}) = -\frac{7}{3}$
 $y - \frac{70}{39} = -\frac{7}{3}$
 $y = -\frac{7}{3} + \frac{70}{39} = -\frac{1}{13}$

$x + \frac{1}{2}(-\frac{1}{13}) - \frac{3}{2}(\frac{35}{13}) = 0$
 $x - \frac{1}{26} - \frac{105}{26} = 0$
 $x = \frac{56}{13}$

($\frac{56}{13}, -\frac{1}{13}, \frac{35}{13}$)

Question #14 continued

c.) $x - y + z = -4$
 $2x - 3y + 4z = -15$
 $5x + y - 2z = 18$

$$\begin{bmatrix} 1 & -1 & 1 & -4 \\ 2 & -3 & 4 & -15 \\ 5 & 1 & -2 & 12 \end{bmatrix} \xrightarrow{-2R_1 + R_2} \begin{bmatrix} 1 & -1 & 1 & -4 \\ 0 & -1 & 2 & -7 \\ 5 & 1 & -2 & 12 \end{bmatrix}$$

$$\xrightarrow{-2R_1 - 2R_2 - 2R_3} \begin{bmatrix} 1 & -1 & 1 & -4 \\ 0 & -1 & 2 & -7 \\ 0 & -1 & -2 & -7 \end{bmatrix}$$

$$\xrightarrow{-5R_1 + R_3} \begin{bmatrix} 1 & -1 & 1 & -4 \\ 0 & -1 & 2 & -7 \\ 0 & 6 & -10 & 32 \end{bmatrix} \xrightarrow{-1R_2} \begin{bmatrix} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 6 & -10 & 32 \end{bmatrix}$$

$$\xrightarrow{-6R_2 + R_3} \begin{bmatrix} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 2 & -10 \end{bmatrix} \xrightarrow{\frac{1}{2}R_3} \begin{bmatrix} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$$\xrightarrow{-6R_2} \begin{bmatrix} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 2 & -10 \end{bmatrix}$$

$$\begin{aligned} x - y + z &= -4 \\ y - 2z &= 7 \\ z &= -5 \end{aligned}$$

$$\begin{aligned} y - 2(-5) &= 7 \\ y + 10 &= 7 \\ y &= -3 \end{aligned}$$

$$\begin{aligned} x - (-3) + (-5) &= -4 \\ x - 2 &= -4 \\ x &= -2 \end{aligned}$$

$(-2, -3, -5)$

Question #15

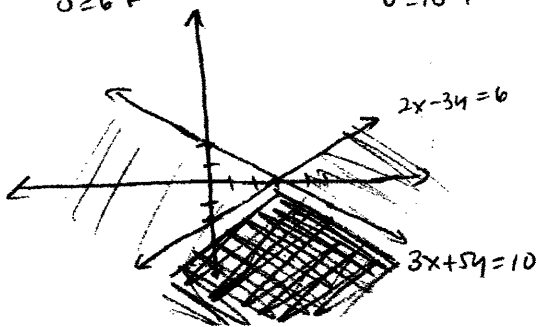
a.) $2x - 3y \geq 6$
 $3x + 5y \leq 10$

$$\begin{aligned} 2x - 3y &\geq 6 \\ -3y &\geq -2x + 6 \\ y &\leq \frac{2}{3}x - 2 \end{aligned}$$

$$\begin{aligned} 3x + 5y &\leq 10 \\ 5y &\leq -3x + 10 \\ y &\leq -\frac{3}{5}x + 2 \end{aligned}$$

$(0,0): 2(0) - 3(0) \geq 6$
 $0 \geq 6$ F

$(0,0): 3(0) + 5(0) \leq 10$
 $0 \leq 10$ T

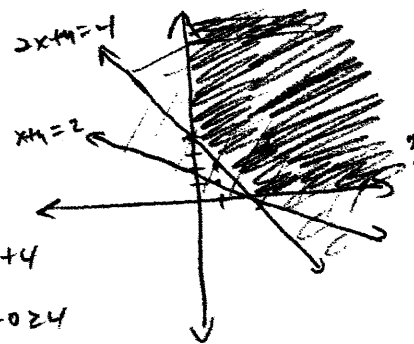


b.) $x \geq 0$
 $y \geq 0$
 $x + y \geq 2$
 $2x + y \geq 4$

$$\begin{aligned} x + y &\geq 2 & 2x + y &\geq 4 \\ y &\geq -x + 2 & y &\geq -2x + 4 \end{aligned}$$

$(0,0): 0 + 0 \geq 2$ F
 $0 \geq 2$ F

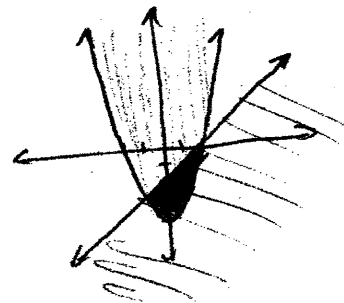
$(0,0): 2(0) + 0 \geq 4$ F
 $0 \geq 4$ F



c.) $y \geq x^2 - 4$
 $y \leq x - 2$

$(0,0): y \geq x^2 - 4$
 $0 \geq 0^2 - 4$
 $0 \geq -4$ T

$(0,0): 0 \leq 0 - 2$
 $0 \leq -2$ F



Question #16

$x \geq 0$
 $y \geq 0$
 $2x + 3y \geq 6$
 $x + y \leq 8$

$z = 3x + 4y$ minimize

$2x + 3y \geq 6$

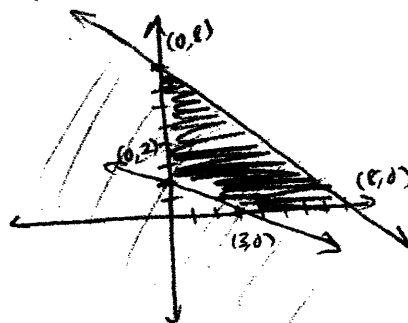
$x + y \leq 8$
 $y \leq -x + 8$

$3y \geq -2x + 6$

$y \geq -\frac{2}{3}x + 2$

$(0,0): 0 \leq 8$ T

$(0,0): 0 \geq 6$ F



$(0,2): z = 3(0) + 4(2) = 8$
 $(3,0): z = 3(3) + 4(0) = 9$
 $(0,8): z = 3(0) + 4(8) = 32$
 $(8,0): z = 3(8) + 4(0) = 24$

**minimum at $(0, 2)$
 minimum value = 8**

Question #17

a) $3A + 2B$

$$3 \begin{bmatrix} 3 & 7 & -2 & 6 \\ -5 & 4 & 1 & 2 \end{bmatrix} + 2 \begin{bmatrix} 4 & -6 & 1 & 2 \\ 8 & 3 & 9 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 21 & -6 & 18 \\ -15 & 12 & 3 & 6 \end{bmatrix} + \begin{bmatrix} 8 & -12 & 2 & 4 \\ 16 & 6 & 18 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 17 & -9 & -4 & 22 \\ 1 & 18 & 21 & 0 \end{bmatrix}$$

b) $2A - B$

$$2 \begin{bmatrix} 3 & 7 & -2 & 6 \\ -5 & 4 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & -6 & 1 & 2 \\ 8 & 3 & 9 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 14 & -4 & 12 \\ -10 & 8 & 2 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 6 & -1 & -2 \\ -8 & -3 & -9 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 20 & -5 & 10 \\ -18 & 5 & -7 & 7 \end{bmatrix}$$

c) $6A$

$$6 \begin{bmatrix} 3 & 7 & -2 & 6 \\ -5 & 4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 18 & 42 & -12 & 36 \\ -30 & 24 & 6 & 12 \end{bmatrix}$$

Question #19

a) $a_n = (-1)^{n+1} (2n+3)$

$a_1 = (-1)^{1+1} (2(1)+3) = 5$

$a_2 = (-1)^{2+1} (2(2)+3) = -7$

$a_3 = (-1)^{3+1} (2(3)+3) = 9$

$a_4 = (-1)^{4+1} (2(4)+3) = -11$

$a_5 = (-1)^{5+1} (2(5)+3) = 13$

b) $b_n = \left\{ \frac{2^n}{n^2} \right\}$

$b_1 = \frac{2^1}{1^2} = 2$

$b_2 = \frac{2^2}{2^2} = 1$

$b_3 = \frac{2^3}{3^2} = \frac{8}{9}$

$b_4 = \frac{2^4}{4^2} = 1$

$b_5 = \frac{2^5}{5^2} = \frac{32}{25}$

Question #18

a) $\begin{bmatrix} 4 & 5 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 6 \\ 7 & -5 & -4 \end{bmatrix}$

$$\begin{bmatrix} (4)(3) + (5)(7) & (4)(-1) + (5)(-5) & (4)(6) + (5)(-4) \\ (-2)(3) + (3)(7) & (-2)(-1) + (3)(-5) & (-2)(6) + (3)(-4) \end{bmatrix}$$

$$\begin{bmatrix} 47 & -29 & 4 \\ 15 & -13 & -24 \end{bmatrix}$$

$2 \times 2 \times 3 = 2 \times 3$

b) $\begin{bmatrix} 3 & 5 & -2 \\ 2 & 3 & -1 \\ -4 & 6 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 6 & 1 \\ 7 & 4 \end{bmatrix}$

$$\begin{bmatrix} (3)(3) + (5)(6) + (-2)(7) & (3)(-2) + (5)(1) + (-2)(4) \\ (2)(3) + (3)(6) + (-1)(7) & (2)(-2) + (3)(1) + (-1)(4) \\ (-4)(3) + (6)(6) + (4)(7) & (-4)(-2) + (6)(1) + (4)(4) \end{bmatrix}$$

$$\begin{bmatrix} 25 & -9 \\ 17 & -5 \\ 52 & 30 \end{bmatrix}$$

$3 \times 3 \times 3 \times 2 = 3 \times 3$

Question #20

a) $\sum_{k=1}^5 (6k^2 + 12) = (1^2 + 12) + (2^2 + 12) + (3^2 + 12) + (4^2 + 12) + (5^2 + 12) = 112$

b) $\sum_{k=1}^9 (-2k + 8)$ arithmetic

$S_9 = \frac{9}{2} [2(6) + (9-1)(-2)] = \frac{9}{2} [12 - 16] = \frac{9}{2} (-4) = -18$

d) $\sum_{k=1}^{40} (3k - 5)$ arithmetic

$S_{40} = \frac{40}{2} [2(-6) + (40-1)(3)] = 2100$

21, e, f, g not on exam #3.

Question #21

a.) $a_7 = 31, a_{20} = 96$

$$a_7 = a_1 + (7-1)d$$

$$31 = a_1 + 6d$$

$$a_{20} = a_1 + (20-1)d$$

$$96 = a_1 + 19d$$

$$\begin{array}{r} 96 = a_1 + 19d \\ -(31 = a_1 + 6d) \\ \hline 65 = 13d \end{array}$$

$$65 = 13d$$

$$d = 5$$

$$31 = a_1 + 6(5)$$

$$31 = a_1 + 30$$

$$a_1 = 1$$

$$a_n = 1 + (n-1)(5)$$

$$a_n = 1 + 5n - 5$$

$$a_n = 5n - 4$$

b.) $a_{12} = 30, a_{22} = 50$

$$a_{12} = a_1 + (12-1)d$$

$$30 = a_1 + 11d$$

$$a_{22} = a_1 + (22-1)d$$

$$50 = a_1 + 21d$$

$$\begin{array}{r} 50 = a_1 + 21d \\ -(30 = a_1 + 11d) \\ \hline 20 = 10d \end{array}$$

$$20 = 10d$$

$$d = 2$$

$$30 = a_1 + 11(2)$$

$$30 = a_1 + 22$$

$$a_1 = 8$$

$$a_n = 8 + (n-1)2$$

$$a_n = 8 + 2n - 2$$

$$a_n = 2n + 6$$