

Math 130 - Final Exam Review Sheet

Please Note: The final exam is comprehensive. The review sheet is designed for you to have a guide as to what to study. The problems on the exam are not limited to the type of problems on this sheet. Any type of problem from the assigned homework problems are possible exam questions. It is also advisable to review your previous exams. More details regarding material previously tested will be given in class. Please attempt other practice problems other than those presented on this sheet in order to be completely prepared for the exam.

1. Solve the following system of equations. Remember that you will need to know how to use the substitution and elimination methods.

$$a. \begin{cases} x - 2y = 16 \\ y + 3 = 3x \end{cases}$$

$$b. \begin{cases} x - 3y = -6 \\ 3x - 9y = 9 \end{cases}$$

$$c. \begin{cases} 2x + 6y = 8 \\ 3x + 9y = 12 \end{cases}$$

$$d. \begin{cases} 3x - 5y = 11 \\ 2x - 6y = 2 \end{cases}$$

2. Solve the following system of equations. Remember that you will need to know how to solve using elimination and using matrices.

$$a. \begin{cases} 3x + 2y - z = 4 \\ 3x - 2y + z = 5 \\ 4x - 5y - z = -1 \end{cases}$$

$$b. \begin{cases} 4x - y + 2z = 11 \\ x + 2y - z = -1 \\ 2x + 2y - 3z = -1 \end{cases}$$

3. Graph the following system of inequalities on the given sheet of graph paper.

$$a. \begin{cases} 2x - 3y \geq 6 \\ 3x + 5y \leq 10 \end{cases}$$

$$b. \begin{cases} x + y \geq 2 \\ 2x + y \geq 4 \end{cases}$$

4. Write the first five terms of the following sequences.

$$a. \{(-1)^{n+1}(2n+3)\}$$

$$b. \left\{ \frac{2^n}{n^2} \right\}$$

$$c. \begin{cases} a_1 = 4 \\ a_n = 3a_{n-1} + 5 \end{cases}$$

5. Evaluate each sum.

$$a. \sum_{k=1}^5 (k^2 + 12)$$

$$b. \sum_{k=1}^9 (-2k + 8)$$

$$c. \sum_{k=1}^7 \left(\frac{1}{3}\right)^k$$

$$d. \sum_{k=1}^{40} (3k - 9)$$

$$e. \sum_{k=1}^{10} 3(-2)^k$$

$$f. \sum_{k=1}^{\infty} 2\left(\frac{1}{2}\right)^{k-1}$$

$$g. \sum_{k=1}^{\infty} 3\left(\frac{5}{4}\right)^{k-1}$$

6. Find a general formula for each arithmetic sequence.

$$a. 7^{\text{th}} \text{ term is } 31; 20^{\text{th}} \text{ term is } 96$$

$$b. 12^{\text{th}} \text{ term is } 30; 22^{\text{nd}} \text{ term is } 50$$

7. Expand each expression using the binomial theorem.

a. $(x+2)^5$

b. $(x-3)^4$

c. $(2x+3)^5$

d. $(3x-4)^4$

8. Evaluate the following.

a. $6!$

b. $0!$

c. $\binom{5}{3}$

d. $\binom{7}{5}$

9. Use the Binomial Theorem to find the indicated coefficient or term.

a. The coefficient of x^6 in the expression $(x+3)^{10}$

b. The coefficient of x^2 in the expression $(2x-3)^9$

c. The third term in the expansion of $(x-3)^7$

d. The sixth term in the expansion of $(3x+2)^8$

10. Given the following functions.

$$f(x) = 3x^2 - 4x + 2$$

$$g(x) = \frac{5}{x-4}$$

$$h(x) = -4x^2 + 2x + 8$$

$$k(x) = \frac{x-3}{x+5}$$

Evaluate:

a. $f(2)$

b. $g(1)$

c. $(f+g)(x)$

d. $(f-h)(2)$

e. $\left(\frac{f}{h}\right)(x)$

f. $f(x-3)$

g. $(g \circ h)(x)$

h. $(h \circ f)(x)$

i. Domain $(f + g)$

j. Domain $(h \cdot k)$

k. Domain $\left(\frac{g}{k}\right)$

l. Domain $(g \circ k)$

11. Find the domains of the following functions.

a. $f(x) = \frac{3}{2x-5}$

b. $f(x) = 2x+1$

c. $f(x) = \frac{7x}{5-x}$

12. Solve the following inequalities.

a. $\frac{x^2 - 4x + 3}{x + 4} \geq 0$

b. $x^2 + 4x < 12$

13. Find all real and imaginary roots of the polynomial equations.

a. $3x^3 - 5x^2 + 2x - 8 = 0$

b. $x^4 - 5x^3 + 3x^2 + 15x - 18 = 0$

c. $2x^3 + 3x^2 + 2x + 3 = 0$

d. $2x^3 - 11x^2 + 10x + 8 = 0$

14. Given $f(x) = \frac{1}{3x+2}$ and $g(x) = x^3 - 3$. Find

a. $f^{-1}(x)$

b. $g^{-1}(x)$

15. Express the following as a sum and/or difference of logarithms.

a. $\log_3 \frac{x^4 y^3}{z^2 w^3}$

b. $\log \sqrt{\frac{x^2}{y^3}}$

16. Express the following sum and difference of logarithms as a single logarithm.

a. $2\log_3 x + 4\log_3 y - 6\log_3 z$

b. $6\log_4 x - 2\log_4 y + 3\log_4 z$

17. Evaluate the following.

a. $\log_5 25$

b. $\log_2 \frac{1}{8}$

c. $\log_7 \sqrt{7}$

d. $\log_5 5$

e. $\log_6 1$

f. $\log_4 4^6$

g. $8^{\log_8 19}$

h. $\log_2 64$

i. $\log_{49} 7$

18. Evaluate the following logarithms by rounding to the nearest thousandth.

a. $\log 52$

b. $\ln 28$

c. $\log_5 12$

d. $\log_7 31$

19. Solve for x .

a. $2^{2x+1} = 4$

b. $3^{x^3} = 9^x$

c. $5^{2x-1} = 125$

$$d. 4^x = \frac{1}{\sqrt{2}}$$

$$e. 3^x = 14$$

$$f. 3^{1-2x} = 4^x$$

$$g. e^{5x} = 7$$

$$h. 9e^x = 99$$

$$i. e^{0.08x} = 4$$

$$j. \log_2(x^2) - \log_2(x-2) = 3$$

$$k. \log_2(x-3) + \log_2(x+4) = 3$$

$$l. x = \log_5 625$$

$$m. \log_8 x = -2$$

$$n. \log_x 4 = \frac{1}{3}$$

$$o. \log_4(x^2 - 9) - \log_4(x+3) = 3$$

$$p. \log(3x-5) - \log 5x = 2$$

20. Find the equation of the line with the given information.

a. Slope = $-\frac{2}{3}$; containing the point $(1, -1)$

b. Containing the points $(-3, 4)$ and $(2, 5)$

c. Slope = -2 ; y-intercept = $(0, -2)$

d. Slope undefined; containing the point $(2, 4)$

21. Find all solutions of the following equations.

a. $2x^2 - 5x - 3 = 0$

b. $x(x+4) = 12$

c. $(2x+3)^2 = 9$

- d. $x^2 - 6x = 13$
- e. $x^2 + \frac{2}{3}x - \frac{1}{3} = 0$
- f. $x^2 + 4x + 2 = 0$
- g. $4x^2 + x + 1 = 0$
- h. $3x = 1 - \frac{1}{x}$

22. Find all solutions of the following equations.

- a. $\sqrt{5t+3} = 2$
- b. $x = 2\sqrt{x-1}$
- c. $2 + \sqrt{12-2x} = x$
- d. $\sqrt{3x+7} + \sqrt{x+2} = 1$
- e. $x^4 - 10x^2 + 25 = 0$
- f. $2(x+1)^2 - 5(x+1) = 3$
- g. $t^{1/2} - 2t^{1/4} + 1 = 0$

23. Given the following functions. Determine the x -intercept(s), y -intercept, vertical asymptote(s), horizontal asymptote, table of values, and the graph.

- a. $f(x) = \frac{3x-2}{x^2-4x+3}$
- b. $f(x) = \frac{3x-4}{x-3}$
- c. $f(x) = \frac{x^2-3x-4}{x+2}$

24. Graph the following functions. For each function: (i) Find the x -intercept(s), (ii) Find the y -intercept, (iii) Use the x -intercept(s) to find the intervals on which the graph of f is above or below the x -axis, (iv) End behavior: find the power function that the graph of f resembles for large values of $|x|$, (v) Determine whether the graph crosses or touches the x -axis at each x -intercept.

- a. $f(x) = (x-2)(x+3)(x-3)$
- b. $f(x) = (x+3)(x-3)(x-1)(x+1)$
- c. $f(x) = (x-1)(x+3)^2(x-2)^3$

25. Find the amount that results from each investment.
- \$50 invested at 6% compounded monthly after a period of 3 years.
 - \$700 invested at 6% compounded daily after a period of 2 years.
 - \$100 invested at 12% compounded continuously after a period of $3\frac{3}{4}$.
26. How many years will it take for an initial investment of \$10,000 to grow to \$25,000? Assume a rate of interest of 6% compounded continuously.
27. David has available 400 yards of fencing and wished to enclose a rectangular area. Express the area A of the rectangle as a function of the width x of the rectangle. For what value of x if the area largest? What is the maximum area?
28. A farmer with 4000 meters of fencing wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area that can be enclosed?
29. The half-life of radium is 1690 years. If 10 grams is present now, how much will be present in 50 years?
30. A piece of charcoal is found to contain 30% of the carbon 14 that it originally had. When did the tree from which the charcoal came die? Use 5600 years as the half-life of carbon 14.
31. The population of a southern city follows the exponential law. If the population doubled in size over an 18-month period and the current population is 10,000, what will the population be 2 years from now?
32. The relationship between Celsius ($^{\circ}C$) and Fahrenheit ($^{\circ}F$) degrees for measuring temperature is linear. Find an equation relating $^{\circ}C$ and $^{\circ}F$ if $0^{\circ}C$ corresponds to $32^{\circ}F$ and $100^{\circ}C$ corresponds to $212^{\circ}F$. Use this equation to find the Celsius measure of $70^{\circ}F$.
33. How many gallons of 50% antifreeze must be mixed with 80 gallons of 20% antifreeze to get a mixture that is 40% antifreeze?
34. How many gallons of a 12% indicator solution must be mixed with a 20% indicator solution to get 10 gallons of a 14% solution?
35. A coin collector has \$1.70 in dimes and nickels. She has 2 more dimes than nickels. How many nickels does she have?
36. Deep Thought Granola is 25% nuts and dried fruit. Oat Dream Granola is 10% nuts and dried fruit. How much of Deep Thought and how much of Oat Dream should be mixed to form a 20-lb. batch of granola that is 19% nuts and dried fruit.

37. A rectangular lot whose perimeter is 360 feet is fenced along three sides. An expensive fencing along the lot's length cost \$20 per foot, and an inexpensive fencing along the two side widths costs only \$8 per foot. The total cost of the fencing along the three sides comes to \$3280. What are the lot's dimensions?

38. The sum of three numbers is 16. The sum of twice the first number, 3 times the second number, and 4 times the third number is 46. The difference between 5 times the first number and the second number is 31. Find the three numbers.

39. Three numbers are unknown. Three times the first number plus the second number plus twice the third number is 5. If 3 times the second number is subtracted from the sum of the first number and 3 times the third number, the result is 2. If the third number is subtracted from 2 times the first number and added to 3 times the second number, the result is 1. Find the numbers.

40. If y varies directly as x , and $y = 9$ when $x = 30$, find y when $x = 40$.

41. If y varies inversely as x , and $y = 20$ when $x = \frac{1}{4}$, find y when $x = 15$.

42. If m varies jointly as z and p , and $m = 10$ when $z = 2$ and $p = 7.5$, find m when $z = 6$ and $p = 9$.

43. Suppose p varies directly as the square of z , and inversely as r . If $p = \frac{32}{5}$ when $z = 4$ and $r = 10$, find p when $z = 3$ and $r = 32$.

44. Given the following matrices:

$$A = \begin{bmatrix} 3 & 7 & -2 & 6 \\ -5 & 4 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -6 & 1 & 2 \\ 8 & 3 & 9 & -3 \end{bmatrix}$$

Determine:

- a. $3A + 2B$
- b. $2A - B$
- c. $6A$

45. Determine the following products.

a. $\begin{bmatrix} 4 & 5 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 6 \\ 7 & -5 & -4 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 5 & -2 \\ 2 & 3 & -1 \\ -4 & 6 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 6 & 1 \\ 7 & 4 \end{bmatrix}$