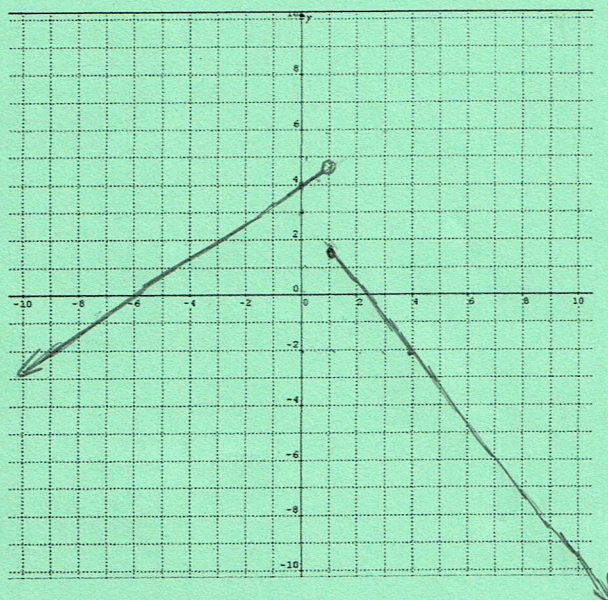


MATH 160 – EXAM #1
Spring Semester 2019

Name: KEY

Directions: Please show all work for maximum credit. No work = no credit. Point values for each problem are given. There is a total of 103 points on this exam. This exam will be taken out of 100 points. Please show all work and clearly indicate your answers. Remember, this exam is to show what you know. You may not use any notes, the textbook, or any unauthorized sources for assistance during this exam. You may use a scientific calculator. Graphing calculators and cell phones may not be used. Clearly indicate the answer to each question. If you have any work on separate paper that you would like graded, you must indicate that on each corresponding problem on this exam. Do not round any numbers on this exam except where appropriate. You will need your Mt. SAC ID to submit this exam. Good luck!

(4 points) 1. Graph the following function on the provided grid: $f(x) = \begin{cases} \frac{2}{3}x + 4 & x < 1 \\ -\frac{5}{4}x + 3 & x \geq 1 \end{cases}$



2. Find the domain of the following functions.

(3 points) a. $f(x) = \sqrt{5x+1}$

$$5x+1 \geq 0$$

$$x \geq -\frac{1}{5}$$

$$\{x \mid x \geq -\frac{1}{5}\} \text{ or } [-\frac{1}{5}, \infty)$$

(3 points) b. $f(x) = \frac{3x-4}{2x^2-5x+3}$

$$2x^2 - 5x + 3 = 0$$

$$(2x - 3)(x - 1) = 0$$

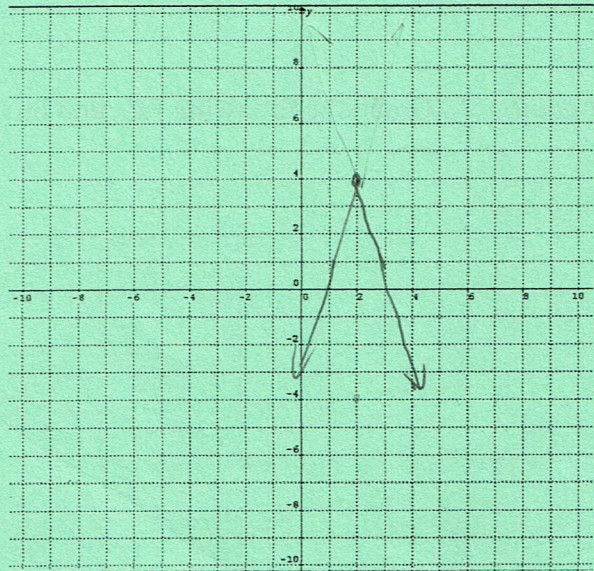
$$x = \frac{3}{2}, 1$$

$$\left\{ x \mid x \neq 1, \frac{3}{2} \right\}$$

3. Sketch the graph of the following functions. Explain how each graph is obtained by using reflections, stretches, compressions, and translations.

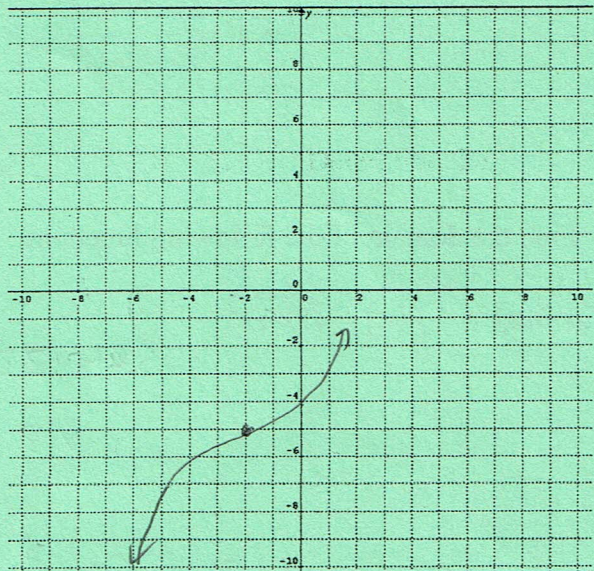
(4 points) a. $f(x) = -3|x-2| + 4$

vertical reflection
stretch by factor of 3
shift 2 units right
shift 4 units up.



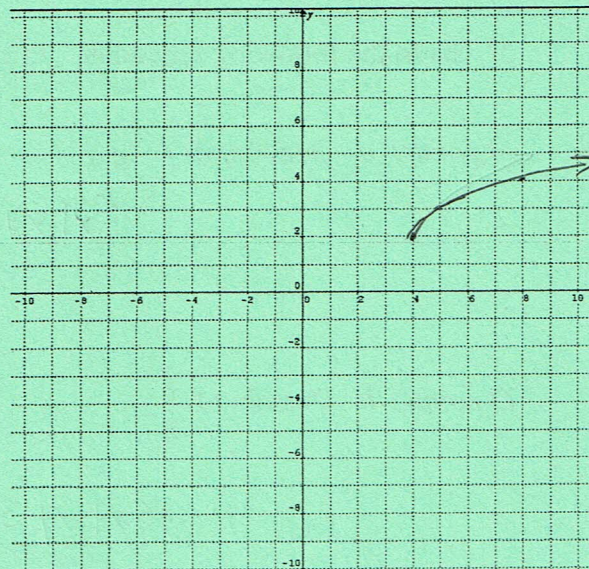
(4 points) b. $k(x) = \frac{1}{2}(x+2)^3 - 5$

compress by factor of $\frac{1}{2}$
shift 2 units left
shift 5 units down.

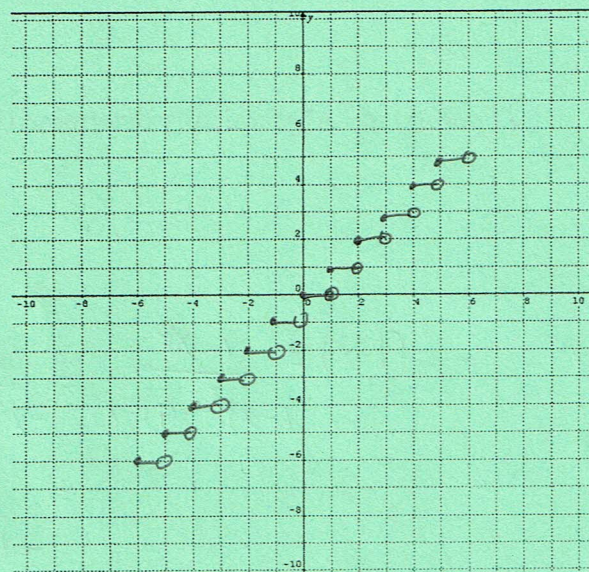


(4 points) c. $k(x) = \sqrt{x-4} + 2$

shift 4 units right
shift 2 units up



(2 points) d. $f(x) = \lfloor x \rfloor$



4. Given the following functions: $f(x) = 2x^2 - 4x + 7$, $g(x) = 5x - 2$. Determine the following.

(3 points) b. $(f-g)(x) = f(x) - g(x)$
 $= (2x^2 - 4x + 7) - (5x - 2)$
 $= 2x^2 - 9x + 9$

(3 points) d. $(f \cdot g)(3) = f(3) \cdot g(3)$
 $= 13 \cdot 13$
 $= 169$

$$f(3) = 2(3)^2 - 4(3) + 7$$

$$= 18 - 12 + 7 = 13$$

$$g(3) = 5(3) - 2 = 13$$

5. Given the following functions: $f(x) = \frac{x-2}{x-3}$, $g(x) = \frac{x+1}{x+2}$. Determine the following.

(3 points) a. Domain of $f - g$.

$$\{x \mid x \neq 3, -2\}$$

(3 points) b. Domain of $\frac{f}{g}$.

$$\{x \mid x \neq 3, -2, -1\}$$

(3 points) 6. Algebraically determine if the following function is even, odd, or neither.

$$f(x) = \frac{x^2}{x^3 - 4x}$$

$$f(-x) = \frac{(-x)^2}{(-x)^3 - 4(-x)} = \frac{x^2}{-x^3 + 4x} \neq f(x) \text{ not even}$$

$$= \frac{x^2}{-(x^3 - 4x)} = -\left(\frac{x^2}{x^3 - 4x}\right) = -f(x) \text{ odd}$$

(4 points) 7. Find the average rate of change of $f(x) = x^3 - 3x^2$ from $x=1$ to $x=4$.

$$f(4) = (4)^3 - 3(4)^2 = 64 - 48 = 16$$

$$f(1) = (1)^3 - 3(1)^2 = 1 - 3 = -2$$

$$\frac{f(4) - f(1)}{4 - 1} = \frac{16 - (-2)}{4 - 1} = \frac{18}{3} = 6$$

(4 points) 8. Find the difference quotient of $f(x) = 3x^2 - 4x$.

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{[3(x+h)^2 - 4(x+h)] - [3x^2 - 4x]}{h} \\ &= \frac{[3(x^2 + 2xh + h^2) - 4x - 4h] - 3x^2 + 4x}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 - 4x - 4h - 3x^2 + 4x}{h} \\ &= \frac{6xh + 3h^2 - 4h}{h} = \frac{h(6x + 3h - 4)}{h} = 6x + 3h - 4 \end{aligned}$$

9. Given $f(x) = 2x^2 + 8x + 6$.

(3 points) a. Determine the vertex of f .

$$x = \frac{-8}{2(2)} = -2$$

$$\begin{aligned} f(-2) &= 2(-2)^2 + 8(-2) + 6 \\ &= 8 - 16 + 6 = -2 \end{aligned}$$

$$(-2, -2)$$

(2 points) d. Determine the y-intercept of f .

$$\begin{aligned} y &= 2(0)^2 + 8(0) + 6 \\ y &= 6 \\ (0, 6) \end{aligned}$$

(2 points) b. Determine the axis of symmetry of f .

$$x = -2$$

(3 points) e. Sketch the graph.

(3 points) c. Determine the x-intercepts of f .

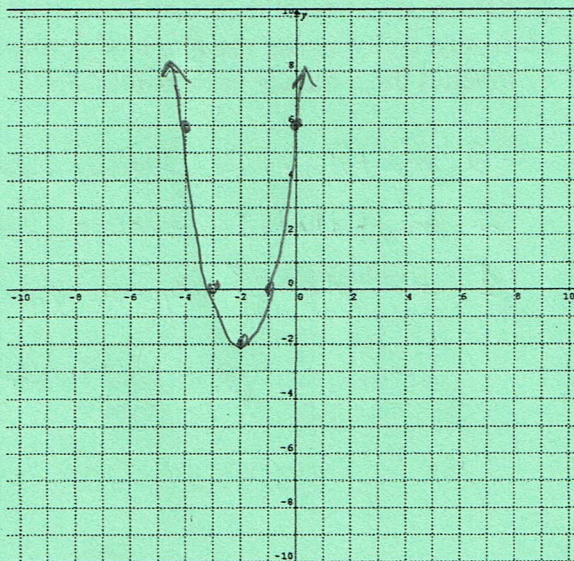
$$2x^2 + 8x + 6 = 0$$

$$2(x^2 + 4x + 3) = 0$$

$$2(x+3)(x+1) = 0$$

$$x = -3, -1$$

$$(-3, 0) (-1, 0)$$



10. Given $f(x) = (x+2)^2(x-2)(x-1)^3$

(2 points) a. Find the x-intercepts of f.

$(-2, 0) (2, 0) (1, 0)$

(1 point) e. Determine the maximum number of turning points.

5

(2 points) b. Find the y-intercept of f.

$$y = (0+2)^2(0-2)(0-1)^3$$

$$= (4)(-2)(-1) = 8$$

$(0, 8)$

(3 points) f. Use the x-intercepts to determine when the graph will be above the x-axis or below the x-axis.

	-2	1	2
$(x+2)^2$	+	+	+
$(x-2)$	-	-	+
$(x-1)^3$	-	-	+
	+	+	-
	above	above	below

(2 points) c. Determine the power function that f resembles as x becomes large (leading term) and draw the arrows that represents the end behavior of f.

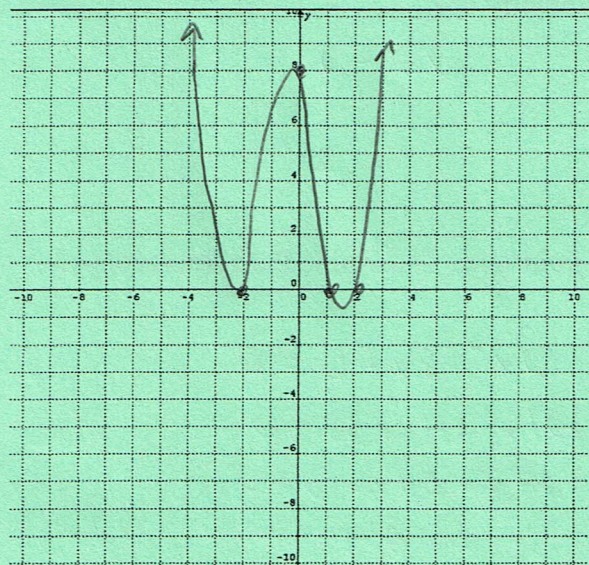
$y = x^6$

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(3 points) g. Using the information obtained from parts a-e, sketch the graph of f.

(3 points) d. Determine the multiplicities of each zero and whether the graph of f crosses or touches the x-axis at each x-intercept.

- 2 multiplicity 2 touches x-axis
- 2 multiplicity 1 crosses x-axis
- 1 multiplicity 3 crosses x-axis



(4 points) 11. Find a polynomial function with zeros $-3, 0, 4$.

$$y = (x+3)(x)(x-4)$$

$$y = x(x^2 - x - 12)$$

$$y = x^3 - x^2 - 12x$$

(1 point each) 12. Given the following graph of a function f .

a. List the number(s) at which f has a local maximum. What are the local maximum values of f ?

local maxima at $x=2$ and $x=6$
local maximum values are $y=5$ and $y=4$

b. List the number(s) at which f has a local minimum. What are the local minimum values of f ?

local minima at $x=-1$ and $x=5$
local minimum values are $y=1$ and $y=3$

c. List the number(s) at which f has an absolute maximum. What is the absolute maximum value of f ?

absolute maximum at $x=-3$
absolute maximum value is $y=7$

d. List the number(s) at which f has an absolute minimum. What is the absolute minimum value of f ?

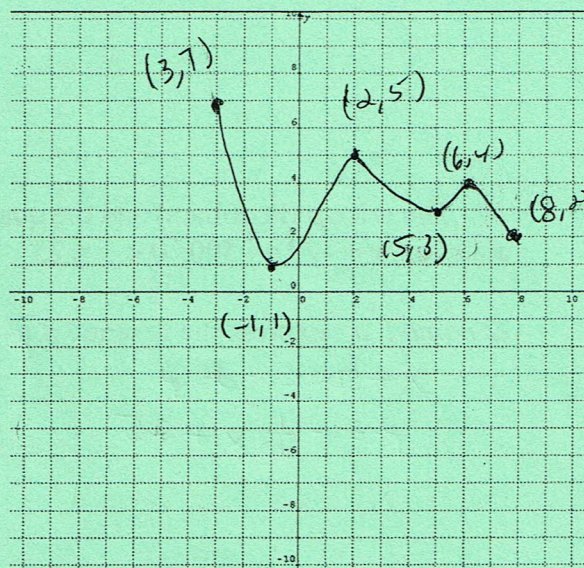
absolute minimum at $x=-1$
absolute minimum value is $y=1$

e. List the interval(s) on which f is increasing.

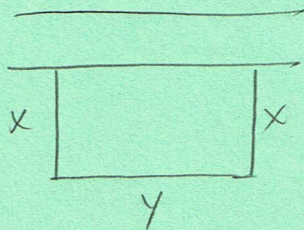
increasing on $(-1, 2) \cup (5, 6)$

f. List the interval(s) on which f is decreasing.

decreasing on $(-3, -1) \cup (2, 5) \cup (6, 8)$



(5 points) 13. You have 600 feet of fencing to enclose a rectangular plot that borders a straight river. If you do not fence to side along the river, find the length and width of the plot that will maximize the area.



$$2x + y = 600$$

$$y = 600 - 2x$$

$$xy = A$$

$$x(600 - 2x) = A$$

$$-2x^2 + 600x = A$$

$$x = -\frac{600}{2(-2)} = 150 \text{ ft}$$

$$y = 600 - 2(150) = 300 \text{ ft}$$

150 ft by 300 ft

14. The price p (in dollars) and the quantity x sold of a certain product obey the demand equation $x = -20p + 500$, $0 < p \leq 25$.

(2 points) a. Express the revenue R as a function of x .

$$x - 500 = -20p$$

$$p = -\frac{1}{20}x + 25$$

$$R = xp$$

$$= x\left(-\frac{1}{20}x + 25\right)$$

$$R = -\frac{1}{20}x^2 + 25x$$

(4 points) b. What quantity x maximizes revenue?

$$x = \frac{-25}{2\left(-\frac{1}{20}\right)} = 250$$

$$x = 250 \text{ units}$$

(2 points) c. What is the maximum revenue?

$$R = -\frac{1}{20}(250)^2 + 25(250)$$

$$R = \$3125$$