MATH 160 – EXAM #2 **Name:** KEY
Spring Semester 2019

Directions: ***Please show all work for maximum credit.*** *No work = no credit.* Point values for each problem are given. There is a total of 102 points on this exam. This exam will be taken out of 100 points. Please show all work and clearly indicate your answers. Remember, this exam is to show what you know. You may not use any notes, the textbook, or any unauthorized sources for assistance during this exam. You may use a scientific calculator. Graphing calculators and cell phones may not be used. Clearly indicate the answer to each question. If you have any work on separate paper that you would like graded, you must be indicate that on each corresponding problem on this exam. Do not round any numbers on this exam except where appropriate. Good luck!

1. Find the horizontal or oblique asymptote of the following functions.

(3 points) a. $f(x)=\dfrac{4-9x^2+6x}{5x+2+8x^2}$ $y=-\frac{9}{8}$

(3 points) b. $f(x)=\dfrac{4x^2-8x+1}{2x-1}$

$$\begin{array}{r} 2x-3 \\ 2x-1\overline{)\,4x^2-8x+1} \\ -(4x^2-2x) \\ \hline -6x+1 \\ -(-6x+3) \\ \hline -2 \end{array}$$

$y=2x-3$

2. Given the following functions: $f(x)=2x^2-4x+7$, $g(x)=5x-2$. Determine the following.

(3 points) a. $(f\circ g)(2)$ $=f(g(2))$ $g(2)=8$
$=f(8)$
$=2(8)^2-4(8)+7$
$=128-32+7=103$

(3 points) b. $(g\circ f)(x)$ $=g(f(x))$
$=g(2x^2-4x+7)$
$=5(2x^2-4x+7)-2$
$=10x^2-20x+35-2$
$=10x^2-20x+33$

3. Given $f(x)=\dfrac{(x-1)}{(x+2)(x-2)}$.

(2 points) *a.* Find the *x*-intercept(s) of *f*.

$(1,0)$

(2 points) *b.* Find the *y*-intercept of *f*.

$y=\dfrac{0-1}{(0+2)(0-2)}=\dfrac{-1}{-4}=\dfrac{1}{4}$

$\left(0,\frac{1}{4}\right)$

(2 points) *c.* Determine the horizontal asymptote.

degree numerator = 1

degree denominator = 2

$y=0$

(2 points) *d.* Determine the vertical asymptote(s).

$(x+2)(x-2)=0$

$x=-2,\ x=2$

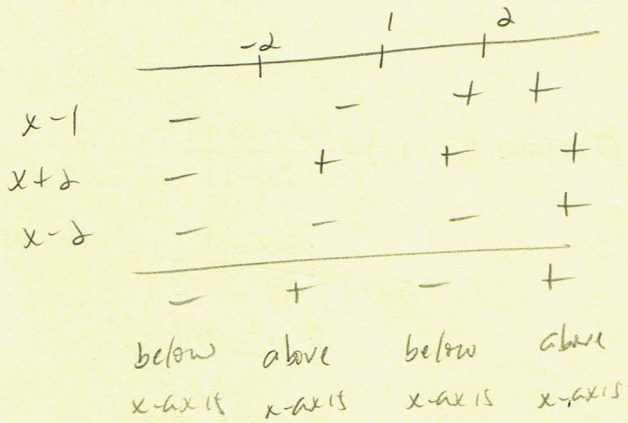
(2 point) *e.* Determine if the graph of the function will intersect the horizontal asymptote.

$\dfrac{x-1}{(x+2)(x-2)}=\dfrac{0}{1}$

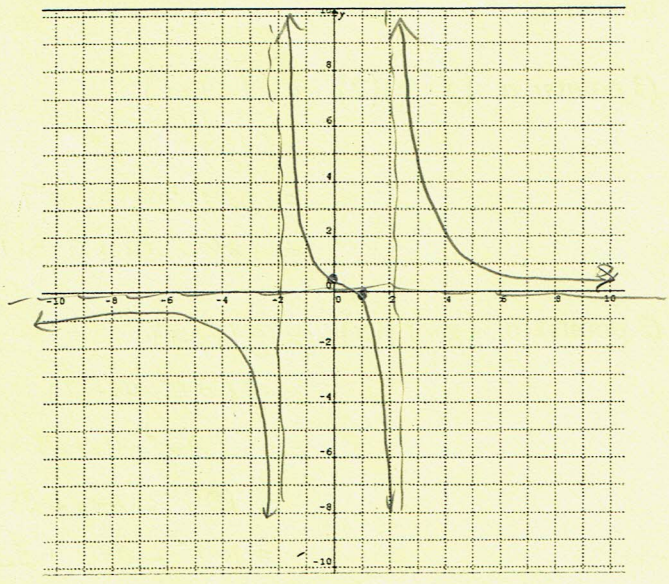
$x-1=0$

$x=1 \qquad (1,0)$

(4 points) *f.* Use the *x*-intercept(s) and the vertical asymptote(s) to determine when the graph will be above the *x*-axis or below the *x*-axis.



(3 points) *g.* Using the information obtained from parts *a-f*, sketch the graph of *f*.



4. Evaluate the following.

(1 point) a. $\log_4 256$ 4

(1 point) b. $\log_{25} 5$ $1/2$

(1 point) c. $\log_7 7$ 1

(1 point) d. $\log_3 3^8$ 8

(1 point) e. $\log_6 1$ 0

(1 point) f. $\log_2 \frac{1}{16}$ -4

(1 points) g. $\log_8 \sqrt[5]{8}$ $1/5$

(1 points) h. $8^{\log_8 73}$ 73

(3 points) 5. Given $f(x)=4x^5-2x^3-6x^2+5x-1$. Use synthetic division to evaluate $f(-2)$.

$$\begin{array}{r|rrrrrr} -2 & 4 & 0 & -2 & -6 & 5 & -1 \\ & & -8 & 16 & -28 & 68 & -146 \\ \hline & 4 & -8 & 14 & -34 & 73 & -147 \end{array}$$

$f(-2) = -147$

(5 points) 6. Given the following function is one-to-one. Find $f^{-1}(x)$.

$$f(x)=\frac{2x-3}{4x+5}$$

$y = \frac{2x-3}{4x+5}$

$x = \frac{2y-3}{4y+5}$

$4xy+5x = 2y-3$

$4xy-2y = -5x-3$

$y(4x-2) = -5x-3$

$y = \frac{-5x-3}{4x-2}$

$f^{-1}(x) = \frac{-5x-3}{4x-2}$

7. Given the following function: $f(x) = 4x^4 - 4x^3 + 17x^2 - 16x + 4$

(3 points) *a.* List all possible rational zeros of the function.

p: $\pm 1, \pm 2, \pm 4$

q: $\pm 1, \pm 2, \pm 4$

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{4}$

(8 points) *b.* Find all the zeros of the function.

$$\begin{array}{r|ccccc} 1/2 & 4 & -4 & 17 & -16 & 4 \\ & & 2 & -1 & 8 & -4 \\ \hline & 4 & -2 & 16 & -8 & 0 \end{array}$$

$$\begin{array}{r|ccc} 1/2 & 4 & -2 & 16 & -8 \\ & & 2 & 0 & 8 \\ \hline & 4 & 0 & 16 & 0 \end{array}$$

$(x - 1/2)^2(4x^2 + 16) = 0$

$4x^2 + 16 = 0$

$x^2 = -4$

$x = \pm 2i$

$x = 1/2, \pm 2i$

(4 points) 8. Use Descartes' Rule of Signs to determine how many positive and how many negative real zeros the polynomial $P(x) = 2x^3 - x^2 + 4x - 7$ can have.

$P(x) = 2x^3 - x^2 + 4x - 7$ 3 sign changes

1 or 3 positive real zeros

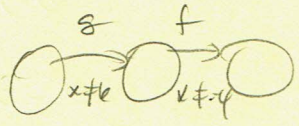
$P(-x) = -2x^3 - x^2 - 4x - 7$ 0 sign changes

0 negative real zeros.

(4 points) 9. Given the following functions: $f(x) = \frac{x-2}{x+4}$, $g(x) = \frac{x+2}{x-6}$. Determine the domain of $(f \circ g)$.

Domain $g = \{x \mid x \neq 6\}$

Domain $f = \{x \mid x \neq -4\}$ set $g(x) = -4$



$\frac{x+2}{x-6} = -\frac{4}{1}$

$x + 2 = -4x + 24$

$5x = 22$

$x = \frac{22}{5}$

Domain $(f \circ g) = \{x \mid x \neq 6, \frac{22}{5}\}$

(4 points) 10. Rewrite the following logarithmic expression as a sum and difference of separate logarithms and exponents as products.

$$\log_5\left(\frac{(x+2)(x-4)^2}{(x-1)^3}\right)$$

$\log_5(x+2) + 2\log_5(x-4) - 3\log_5(x-1)$

(4 points) 11. Condense the following sum and difference of separate logarithms into a single logarithm.

$$3\log_4 x - 7\log_4 y + 4\log_4 z$$

$\log_4 \frac{x^3 z^4}{y^7}$

12. Solve the following equations.

(4 points) a. $4^{3x-2} = 2^{2x+5}$

$(2^2)^{3x-2} = 2^{2x+5}$

$2^{6x-4} = 2^{2x+5}$

$6x-4 = 2x+5$

$4x = 9$

$x = 9/4$

(4 points) b. $\log_3(3x-2) = 2$

$3^2 = 3x-2$

$9 = 3x-2$

$11 = 3x$

$x = 11/3$

(4 points) 13. Use composition of functions to verify that $f(x)=2x-6$ and $g(x)=\frac{1}{2}x+3$ are inverses.

$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{2}x+3\right) = 2\left(\frac{1}{2}x+3\right)-6 = x+6-6 = x$

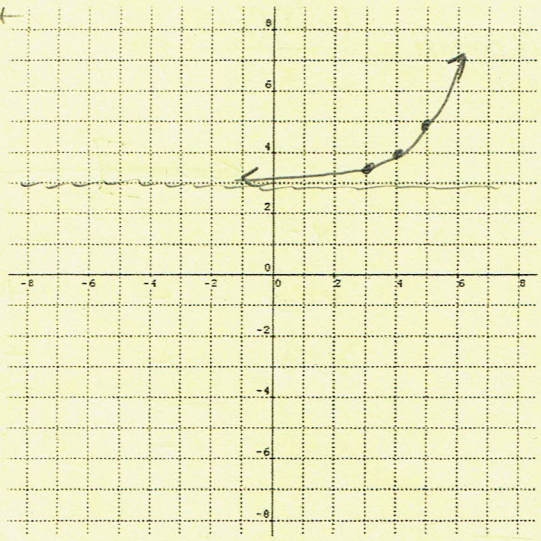
$(g \circ f)(x) = g(f(x)) = g(2x-6) = \frac{1}{2}(2x-6)+3 = x-3+3 = x$

So, f and g are inverses.

14. Sketch the graph of the following functions. *Explain how each graph is obtained by using reflections, stretches, compressions, and translations.*

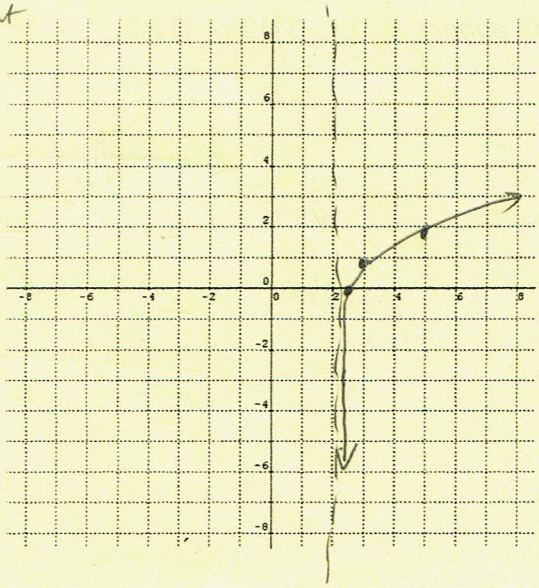
(3 points) a. $f(x)=3+2^{x-4}$ shift 4 units right, shift 3 units up

$y=2^x$	$y=3+2^{x-4}$
$(-1, \frac{1}{2})$	$(3, 3\frac{1}{2})$
$(0,1)$	$(4,4)$
$(1,2)$	$(5,5)$
$\uparrow +4$	$\uparrow +3$



(3 points) b. $f(x)=1+\log_3(x-2)$ shift 2 units right, shift 1 unit up

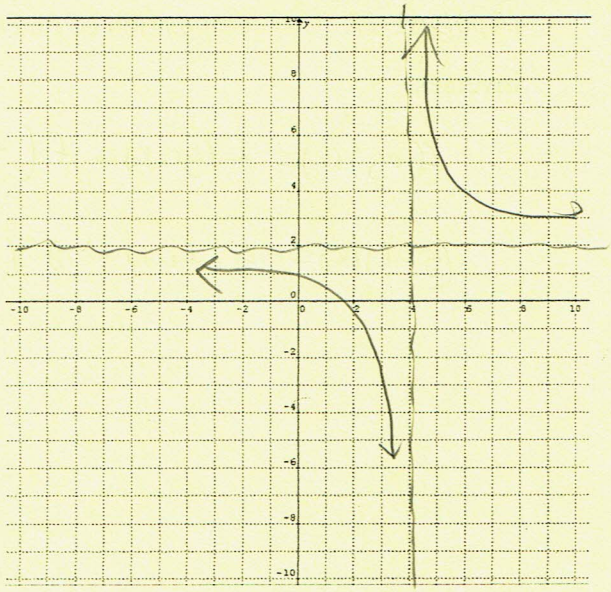
$y=3^x$	$y=\log_3 x$	$y=1+\log_3(x-2)$
$(-1, \frac{1}{3})$	$(\frac{1}{3}, -1)$	$(2\frac{1}{3}, 0)$
$(0,1)$	$(1,0)$	$(3,1)$
$(1,3)$	$(3,1)$	$(5,2)$
	$\uparrow +2$ $\uparrow +1$	



(4 points) c. $f(x)=\dfrac{1}{x-4}+2$

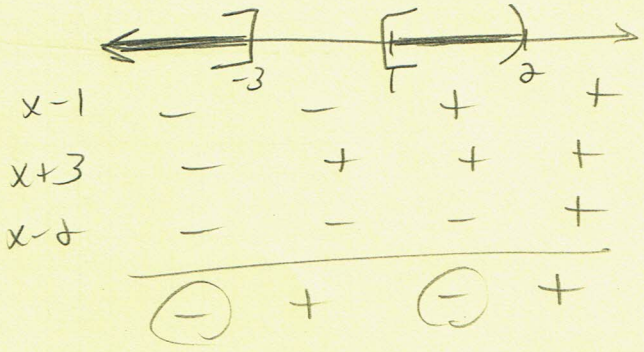
shift 4 units right

shift 2 units up



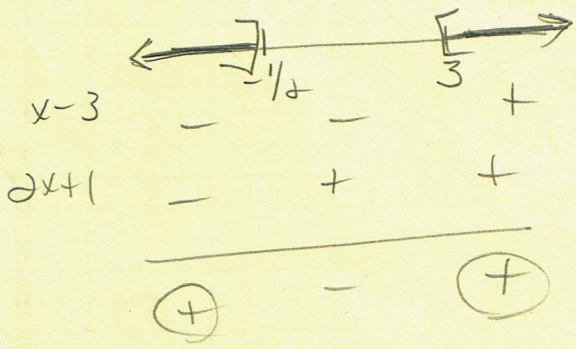
15. Solve the following inequalities. Write your answer in interval notation.

(4 points) a. $\dfrac{(x-1)(x+3)}{x-2}\le 0$



$(-\infty,-3]\cup[1,2)$

(4 points) b. $(x-3)(2x+1)\ge 0$



$(-\infty,-\tfrac{1}{2}]\cup[3,\infty)$