

MATH 160 – EXAM #3
NO CALCULATOR PART
Spring Semester 2019

Name: KEY

Directions: Please show all work for maximum credit. No work = no credit. Point values for each problem are given. There is a total of 102 points on both parts of this exam. The entire exam will be taken out of 100 points. There are 23 points on this part of the exam. Please show all work and clearly indicate your answers. Remember, this exam is to show what you know. You may not use any notes, the textbook, or any unauthorized sources for assistance during this exam. You may not use a calculator on this part of the exam. Clearly indicate the answer to each question. Any work on separate paper that you would like graded must be indicated on each corresponding problem on this exam. Good luck!

(1 point each) 1. Find the exact value of the following trigonometric expressions.

a. $\sin 240^\circ = -\frac{\sqrt{3}}{2}$

b. $\sec 315^\circ = \sqrt{2}$

c. $\tan 450^\circ = \text{undefined}$

d. $\cos \frac{13\pi}{6} = \frac{\sqrt{3}}{2}$

e. $\cot \pi = \text{undefined}$

f. $\csc\left(-\frac{\pi}{3}\right) = -\frac{2}{\sqrt{3}}$

2. Find the exact value of the following trigonometric expressions.

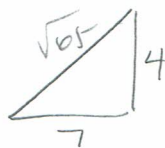
(1 point) a. $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$ or -60°

(1 points) b. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ or -60°

(2 points) c. $\sin\left(\sin^{-1}\frac{5}{4}\right)$ undefined

(2 points) d. $\sin\left(\tan^{-1}\frac{4}{7}\right)$

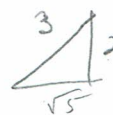
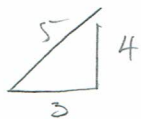
$$= \frac{4}{\sqrt{65}}$$



$$\begin{aligned} 4^2 + 7^2 &= c^2 \\ 16 + 49 &= c^2 \\ 65 &= c^2 \\ \sqrt{65} &= c \end{aligned}$$

(2 points) e. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \frac{5\pi}{6}$ or 150°

(4 points) f. $\sin\left(\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{2}{3}\right)$



$$= \sin\left(\cos^{-1}\frac{3}{5}\right) \cos\left(\sin^{-1}\frac{2}{3}\right) + \cos\left(\cos^{-1}\frac{3}{5}\right) \sin\left(\sin^{-1}\frac{2}{3}\right)$$

$$= \frac{4}{5} \cdot \frac{\sqrt{5}}{3} + \frac{3}{5} \cdot \frac{2}{3} = \frac{4\sqrt{5} + 6}{15}$$

(5 points) 3. Find the exact value of $\sin 15^\circ$ by using a trigonometric identity.

$$\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

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Name: KEY

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1. Solve the following equations.

(4 points) a. $4^{x^2} = 2^x$

$$(2^2)^{x^2} = 2^x$$

$$2^{2x^2} = 2^x$$

$$2x^2 = x$$

$$2x^2 - x = 0$$

$$x(2x-1) = 0$$

$$x = 0 \quad 2x - 1 = 0$$

$$x = 0, \frac{1}{2}$$

(4 points) b. $\log_3(3x-2) = 2$

$$3^2 = 3x-2$$

$$9 = 3x-2$$

$$11 = 3x$$

$$x = \frac{11}{3}$$

check:

$$\log_3\left(3 \cdot \frac{11}{3} - 2\right) = 2$$

$$\frac{11-2}{9} \checkmark$$

$$x = \frac{11}{3}$$

(4 points) c. $3^{2x-5} = 5^{4x}$

$$\ln 3^{2x-5} = \ln 5^{4x}$$

$$(2x-5) \ln 3 = 4x \ln 5$$

$$2x \ln 3 - 5 \ln 3 = 4x \ln 5$$

$$-5 \ln 3 = 4x \ln 5 - 2x \ln 3$$

$$-5 \ln 3 = x(4 \ln 5 - 2 \ln 3)$$

$$x = \frac{-5 \ln 3}{4 \ln 5 - 2 \ln 3}$$

$$4 \ln 5 - 2 \ln 3$$

(4 points) d. $\log(x+21) + \log x = 2$

$$\begin{aligned}\log x(x+21) &= 2 \\ 10^2 &= x(x+21) \\ 100 &= x(x+21) \\ 100 &= x^2 + 21x \\ 0 &= x^2\end{aligned}$$

(2 points) 2. Find an approximate value of $\log_5 42$. Round your answer to the nearest thousandth.

$$\frac{\ln 42}{\ln 5} \approx 2.322$$

(2 points) 3. Find the amount that results if you invest \$1200 at 4% compounded monthly after a period of 5 years.

$$A = 1200 \left(1 + \frac{0.04}{12}\right)^{12(5)}$$

$$A = \$1465.20$$

(2 points) 4. Find the amount that results if you invest \$3000 at 6% compounded continuously after a period of 8 years.

$$A = 3000 e^{0.06(8)}$$

$$A = \$4848.22$$

(4 points) 5. The population of a city follows the exponential law. If the population doubled in size over an 24-month period and if the current population is 25,000, what is the population of the city in 3 years?

$$A = A_0 e^{kt}$$

$$A = 25000 e^{kt}$$

$$50000 = 25000 e^{k(24)}$$

$$2 = e^{24k}$$

$$\ln 2 = \ln e^{24k}$$

$$\ln 2 = 24k$$

$$k = \frac{\ln 2}{24} \approx 0.02888$$

$$A = 25000 e^{0.02888t}$$

$$A = 25000 e^{0.02888(36)}$$

$$A \approx 70708 \text{ people.}$$

(4 points) 6. A fossilized leaf contains 70% of its normal amount of carbon-14. Using the fact that the half-life of carbon-14 is 5730 years, how old is the fossil? Round your answer to the nearest year.

$$A = A_0 e^{kt}$$

$$\frac{0.5 A_0}{A_0} = \frac{A_0 e^{k(5730)}}{A_0}$$

$$0.5 = e^{5730k}$$

$$\ln 0.5 = \ln e^{5730k}$$

$$\ln 0.5 = 5730k$$

$$k = \frac{\ln 0.5}{5730} \approx -0.000120968$$

$$A = A_0 e^{-0.000120968t}$$

$$\frac{0.7 A_0}{A_0} = \frac{A_0 e^{-0.000120968t}}{A_0}$$

$$0.7 = e^{-0.000120968t}$$

$$\ln 0.7 = \ln e^{-0.000120968t}$$

$$\ln 0.7 = -0.000120968t$$

$$t = \frac{\ln 0.7}{-0.000120968} \approx 2949 \text{ yrs.}$$

(6 points) 7. For the following functions, determine the period, phase shift, amplitude, and vertical translation. Then, sketch the graph of one period for each function.

$$y = 2 + 3 \cos\left(2x - \frac{\pi}{4}\right)$$

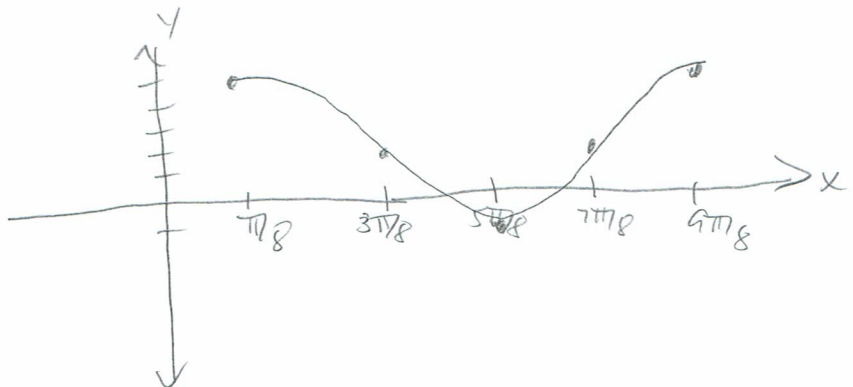
$$y = 2 + 3 \cos 2\left(x - \frac{\pi}{8}\right)$$

amplitude = 3
 period = $\frac{2\pi}{2} = \pi$
 phase shift = $\frac{\pi}{8}$
 vertical shift = 2

original period: $[0, \pi]$

shifted period: $[\frac{\pi}{8}, \frac{9\pi}{8}]$

$\frac{\pi}{4}, \frac{1}{4} = \frac{\pi}{4}$ steps

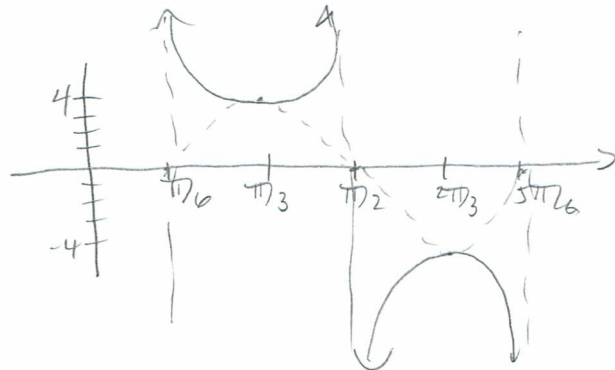


(6 points) 8. Determine the period and phase shift for the following function. Then, sketch the graph of one period for the function.

$$y = 4 \csc\left(3x - \frac{\pi}{2}\right) \quad \text{period} = \frac{2\pi}{3}$$

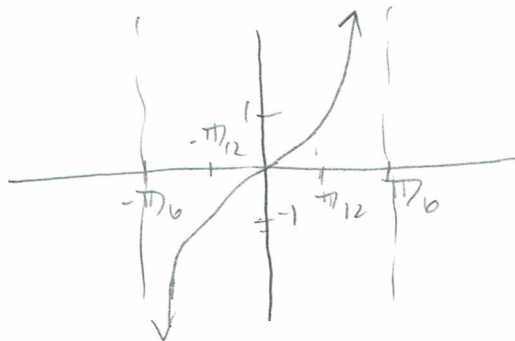
$$y = 4 \csc 3\left(x - \frac{\pi}{6}\right) \quad \text{phase shift} = \frac{\pi}{6}$$

original period: $[0, 2\pi/3]$
 shifted period: $[\pi/6, 5\pi/6]$



(3 points) 9. Determine the period of the following function. Then, sketch the function.

$$f(x) = \tan 3x \quad \text{period} = \frac{\pi}{3}$$



10. Solve the following trigonometric equations on the interval $0 \leq x < 2\pi$.

(6 points) a. $2 \sin^2 x = 1 + \cos x$

$$2(1 - \cos^2 x) = 1 + \cos x$$

$$2 - 2\cos^2 x = 1 + \cos x$$

$$0 = 2\cos^2 x + \cos x - 1$$

$$0 = (2\cos x - 1)(\cos x + 1)$$

$$2\cos x - 1 = 0 \quad \cos x + 1 = 0$$

$$\cos x = 1/2 \quad \cos x = -1$$

$$x = 60^\circ, 300^\circ \quad x = 180^\circ$$

(6 points) b. $\cos(3x) = -\frac{\sqrt{3}}{2}$ $3x = 150^\circ + 360^\circ k$ $3x = 210^\circ + 360^\circ k$, k is an integer
 $x = 50^\circ + 120^\circ k$ $x = 70^\circ + 120^\circ k$

$k=0$ $x = 50^\circ$ $x = 70^\circ$
 $k=1$ $x = 170^\circ$ $x = 190^\circ$
 $k=2$ $x = 290^\circ$ $x = 310^\circ$

11. Verify the following trigonometric identities:

(5 points) a. $\frac{\csc \theta - 1}{\cot \theta} = \frac{\cot \theta}{\csc \theta + 1}$

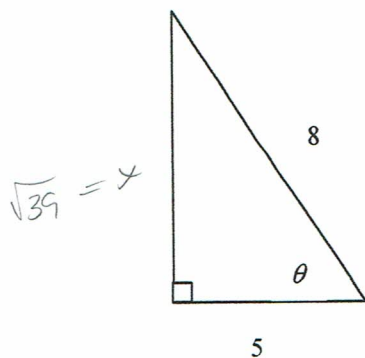
$$\frac{\csc \theta - 1}{\cot \theta} \cdot \frac{\csc \theta + 1}{\csc \theta + 1} = \frac{\csc^2 \theta - 1}{\cot \theta (\csc \theta + 1)} = \frac{\cot^2 \theta}{\cot \theta (\csc \theta + 1)} = \frac{\cot \theta}{\csc \theta + 1}$$

(5 points) b. $\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \sin^2 \theta - \cos^2 \theta$

$$\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}} = \frac{\frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta}}{\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}}$$

$$= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta} \cdot \frac{\sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta} = \sin^2 \theta - \cos^2 \theta$$

(6 points) 12. Given the following right triangle.



$$\begin{aligned}x^2 + 5^2 &= 8^2 \\x^2 + 25 &= 64 \\x^2 &= 39 \\x &= \sqrt{39}\end{aligned}$$

Find the exact value of the six trigonometric ratios of θ .

i. $\csc\theta = \frac{8}{\sqrt{39}}$

iv. $\sec\theta = \frac{8}{5}$

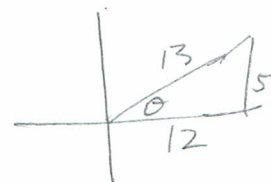
ii. $\cot\theta = \frac{5}{\sqrt{39}}$

v. $\tan\theta = \frac{\sqrt{39}}{5}$

iii. $\sin\theta = \frac{\sqrt{39}}{8}$

vi. $\cos\theta = \frac{5}{8}$

13. Given $\sin\theta = \frac{5}{13}$ with θ in Quadrant I, determine the following:



(3 points) a. $\sin(2\theta) = 2\sin\theta\cos\theta$
 $= 2 \cdot \frac{5}{13} \cdot \frac{12}{13} = \frac{120}{169}$

(3 points) b. $\cos(2\theta) = 1 - 2\sin^2\theta$
 $= 1 - 2\left(\frac{5}{13}\right)^2$
 $= 1 - 2\left(\frac{25}{169}\right)$
 $= 1 - \frac{50}{169} = \frac{169-50}{169} = \frac{119}{169}$