Math 160 - Exam #3 Review Sheet

Please Note: The exam will cover 5.3 to 7.6. The review sheet is designed for you to have a guide as to what to study. The problems on the exam are not limited to the type of problems on this sheet. Any type of problem from the assigned homework problems are possible exam questions. Please remember to know all your trigonometric identities. Please attempt other practice problems other than those presented on this sheet in order to be completely prepared for the exam.

1. Solve for *x*:

a. $2^{2x+1} = 4$
b. $3^{x^3} = 9^x$
c. $3^x = 14$
d. $3^{1-2x} = 4^x$
e. $\log_2\left(x^2\right) - \log_2\left(x-2\right) = 3$
f. $\log_2\left(x-3\right) + \log_2\left(x+4\right) = 3$
g. $x = \log_5 625$
h. $\log_8 x = -2$
i. $\log_x 4 = \frac{1}{3}$
j. $\log_4\left(x^2-9\right) - \log_4\left(x+3\right) = 3$

2. Find the amount that results from each investment.

a. \$50 invested at 6% compounded monthly after a period of 3 years.
b. \$700 invested at 6% compounded daily after a period of 2 years.
c. \$100 invested at 12% compounded continuously after a period of $3\frac{3}{4}$.

3. Find the principal needed now to get each amount; that is, fond the present value.

a. To get \$75 after 3 years at 8% compounded quarterly.
b. To get \$300 after 4 years at 3% compounded daily.
c. To get \$800 after $2\frac{1}{2}$ years at 8% compounded continuously.

4. How many years will it take for an initial investment of \$10,000 to grow to \$25,000? Assume a rate of interest of 6% compounded continuously.

5. Tanya has just inherited a diamond ring appraised at \$5000. If diamonds have appreciated in value at an annual rate of 8%, what was the value of the ring 10 years ago when the ring was purchased?

6. The number N of bacteria present in a culture at a time t (in hours) obeys the function $N(t) = 1000e^{0.01t}$. After how many hours will the population equal 1500? After how many hours will the population equal 2000?

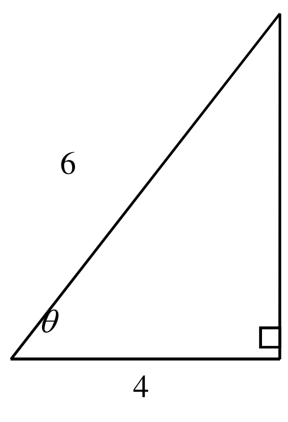
7. Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). What is the half-life of iodine 131? Determine how long it would take for 100 grams of iodine 131 to decay to 10 grams.

8. The population of a southern city follows the exponential law. If the population doubled in size over an 18-month period and the current population is 10,000, what will the population be 2 years from now?

9. The half-life of radium is 1690 years. If 10 grams is present now, how much will be present in 50 years?

10. A piece of charcoal is found to contain 30% of the carbon 14 that it originally had. When did the tree from which the charcoal came die? Use 5600 years as the half-life of carbon 14.

11. Solve the following right triangle. Then, find the six trigonometric ratios of θ in the following triangle:



12. Convert $160°$ to radians.

13. Convert $\frac{7\pi}{12}$ to degrees.

14. Find the exact value of the following trigonometric expressions:

a. $\tan 450°$
b. $\sec 225°$
c. $\cos(-150°)$

d. $\sin 1080°$

e. $\sin\left(-\frac{5\pi}{6}\right)$

f. $\tan\frac{7\pi}{4}$

g. $\csc\left(-\frac{13\pi}{6}\right)$

h. $\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

i. $\sin^{-1}\frac{1}{\sqrt{2}}$

j. $\cos^{-1}(-1)$

k. $\sin\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$

l. $\tan\left(\cos^{-1}\frac{4}{5}\right)$

m. $\sin\left(\cos^{-1}\frac{13}{12}\right)$

n. $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$

15. Determine the amplitude, period and phase shift of the following trigonometric functions. Sketch the graph of the function for one period.

a. $y = 2\sin 3\left(x - \frac{\pi}{6}\right)$

b. $y = 3\cos\left(2x - \frac{\pi}{2}\right)$

c. $y = -4\sin\left(\frac{x}{2} + \frac{\pi}{2}\right)$

16. Determine the period and phase shift of the following trigonometric functions. Sketch the graph of the function for one period.

a. $y = 3\tan(2x)$

b. $y = 4\cot(3x)$

c. $y = 2\csc\left(3x - \frac{\pi}{2}\right)$

d. $y = 4\sec\left(2x - \frac{\pi}{4}\right)$

17. Solve for x in the interval $0 \le x < 2\pi$:

a. $2\sin^2 x - 1 = \sin x$

b. $\cos 2x - \cos x = 0$

c. $\sin x(\cos x + 1) = 0$

d. $\sec^2 x = 2\tan x$

e. $\cos(2\theta) = 2 - 2\sin^2\theta$

f. $\cos(2\theta) + \cos(4\theta) = 0$

g. $\csc^2\theta = \cot\theta + 1$

h. $\sqrt{3}\sin\theta + \cos\theta = 1$

18. Verify the following identities:

a. $\tan^2\theta - \sin^2\theta = \sin^2\theta\tan^2\theta$

b. $\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{1}{\sin^2\theta - \sin^4\theta}$

c. $\frac{\sin\left(\theta + \frac{\pi}{2}\right)}{\cos\left(\theta + \frac{\pi}{2}\right)} = -\cot\theta$

d. $\sin^2\frac{x}{2} = \frac{\sec x - 1}{2\sec x}$

e. $\cos 2x = \cos^4 x - \sin^4 x$

f. $\dfrac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos(2\theta)$

g. $\sin^2\theta\cos^2\theta = \dfrac{1}{8}\left[1 - \cos(4\theta)\right]$

h. $\dfrac{\sin(\alpha - \beta)}{\sin\alpha\cos\beta} = 1 - \cot\alpha\tan\beta$

i. $\dfrac{\csc\theta}{1 - \cos\theta} = \dfrac{1 + \cos\theta}{\sin^3\theta}$

19. Solve the following equations on the interval $0 \le \theta < 2\pi$.

a. $2\cos\theta + \sqrt{2} = 0$

b. $\cos(2\theta) = \sin\theta$

c. $2\sin^2\theta - 3\sin\theta + 1 = 0$

d. $4\sin^2\theta = 1 + 4\cos\theta$

e. $\cos(3\theta) = -1$

20. A right triangle has a hypotenuse of length 10 centimeters. If one angle is $40°$, find the length of each leg.