Math 160 - Final Exam Review Sheet

Please Note: The final exam is comprehensive. The review sheet is designed for you to have a guide as to what to study. The problems on the exam are not limited to the type of problems on this sheet. Any type of problem from the assigned homework problems are possible exam questions. Please remember to know all your trigonometric identities. It is also advisable to review your previous exams. Please attempt other practice problems other than those presented on this sheet in order to be completely prepared for the exam.

1. Identify each equation. If it is a parabola, give its vertex, focus, and directrix. If it is an ellipse, give its center, foci, and vertices. If it is a hyperbola, give its center, vertices, foci, and asymptotes. Then, graph each equation.

a. $x = 2(y-3)^2 + 1$

b. $(x-3)^2 + (y+4)^2 = 36$

c. $\dfrac{(x-3)^2}{16} + \dfrac{(y-5)}{9} = 1$

d. $\dfrac{(x-2)^2}{9} - \dfrac{(y+1)^2}{4} = 1$

e. $\dfrac{(y-4)^2}{25} - \dfrac{(x-3)^2}{36} = 1$

2. Write the partial fraction decomposition of each rational expression.

a. $\dfrac{3x}{(x+2)(x-1)}$

b. $\dfrac{7x+3}{x^3 - 2x^2 - 3x}$

c. $\dfrac{x^2}{(x-1)^2(x+1)}$

d. $\dfrac{x+4}{x^2(x^2+4)}$

e. $\dfrac{x^2+2x+3}{(x^2+4)^2}$

3. Find an equation of a line that is perpendicular to $2x + y = 2$ and containing the point $(-3, 0)$.

4. Find an equation of a line parallel to $y = 2x$ and containing the point $(-1, 2)$.

5. Solve the following inequalities.

a. $\dfrac{x^2 - 4x + 3}{x + 4} \geq 0$

b. $x^2 + 4x < 12$

6. Find all real roots of $x^4 - 5x^3 + 3x^2 + 15x - 18 = 0$.

7. Given $f(x) = \dfrac{1}{3x + 2}$ and $g(x) = x^2 - 3$. Find

a. $f^{-1}(x)$

b. $g^{-1}(x)$ on $[0, \infty)$

8. Solve for *x*:

a. $3^{x^3} = 9^x$

b. $3^{2x} + 3^{x+1} - 4 = 0$

c. $3^{1-2x} = 4^x$

d. $\log_2(x^2) - \log_2(x - 2) = 3$

e. $\log_2(x - 3) + \log_2(x + 4) = 3$

9. Find the exact value of the following trigonometric expressions:

a. $\cos(-150°)$

b. $\sin 1080°$

c. $\sin\left(-\dfrac{5\pi}{6}\right)$

d. $\csc\left(-\dfrac{13\pi}{6}\right)$

e. $\sin^{-1}\dfrac{1}{\sqrt{2}}$

f. $\tan\left(\cos^{-1}\frac{4}{5}\right)$

g. $\sin\left(\cos^{-1}\frac{13}{12}\right)$

h. $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$

10. Determine the amplitude, period and phase shift of the following trigonometric functions. Sketch the graph of the function for one period.

a. $y = 2\sin 3\left(x - \frac{\pi}{6}\right)$

b. $y = -4\cos\left(\frac{x}{2} + \frac{\pi}{2}\right)$

11. Determine the period and phase shift of the following trigonometric functions. Sketch the graph of the function for one period.

a. $y = 3\tan(2x)$

b. $y = 4\cot(3x)$

c. $y = 3\csc\left(2x - \frac{\pi}{4}\right)$

d. $y = 3\sec\left(3x - \frac{\pi}{2}\right)$

12. Solve for x in the interval $0 \le x < 2\pi$:

a. $2\sin^2 x - 1 = \sin x$
b. $\cos 2x - \cos x = 0$
c. $\sec^2 x = 2\tan x$
d. $\cos(2\theta) = 2 - 2\sin^2\theta$
e. $\csc^2\theta = \cot\theta + 1$

13. Verify the following identities:

a. $\tan^2\theta - \sin^2\theta = \sin^2\theta\tan^2\theta$

b. $\dfrac{1}{\cos^2\theta}+\dfrac{1}{\sin^2\theta}=\dfrac{1}{\sin^2\theta-\sin^4\theta}$

c. $\dfrac{\sin\left(\theta+\dfrac{\pi}{2}\right)}{\cos\left(\theta+\dfrac{\pi}{2}\right)}=-\cot\theta$

d. $\sin^2\dfrac{x}{2}=\dfrac{\sec x-1}{2\sec x}$

e. $\cos 2x=\cos^4 x-\sin^4 x$

14. Given the following functions. Determine the *x*-intercept(s), *y*-intercept, vertical asymptote(s), horizontal asymptote, table of values, and the graph.

a. $f(x)=\dfrac{3x-2}{x^2-4x+3}$

b. $f(x)=\dfrac{3x-4}{x-3}$

15. Graph the following functions. For each function: *(i)* Find the *x*-intercept(s), *(ii)* Find the *y*-intercept, *(iii)* End behavior: find the power function that the graph of *f* resembles for large values of $|x|$, *(iv)* Determine whether the graph crosses or touches the *x*-axis at each *x*-intercept, *(v)* Determine the maximum number of turning points, *(vi)* Use the *x*-intercept(s) to find the intervals on which the graph of *f* is above or below the *x*-axis,.

a. $f(x)=-3(x-7)(x+3)^2$

b. $f(x)=x^2(x-3)(x+1)$

16. Graph the following functions. Describe any transformations, stretches, compressions and reflections.

a. $f(x)=\sqrt{x+3}-4$

b. $f(x)=\dfrac{1}{x-3}+2$

c. $f(x) = [\![x]\!]$

d. $f(x) = |x+5| - 2$

17. Determine the domain of the following functions.

a. $f(x) = \dfrac{2x}{x^2 - 16}$

b. $g(x) = \sqrt{3x-5}$

c. $f(x) = \dfrac{x-2}{x^3 - x}$

d. $g(x) = \dfrac{x}{\sqrt{x-6}}$

18. Given $f(x) = \sqrt{4x+1}$, $g(x) = x^2 - 1$, $h(x) = \dfrac{3}{x-4}$ and $k(x) = 3x - 5$. Determine the following.

a. $(f+g)(x)$

b. $(g+k)(x)$

c. $(g \cdot h)(x)$

d. $\left(\dfrac{g}{k}\right)(x)$

e. Domain $(f+h)$

f. Domain $\left(\dfrac{h}{g}\right)$

19. How many years will it take for an initial investment of $10,000 to grow to $25,000? Assume a rate of interest of 6% compounded continuously.

20. A right triangle has a hypotenuse of length 10 centimeters. If one angle is $40°$, find the length of each leg.

21. David has available 400 yards of fencing and wished to enclose a rectangular area. Express the area *A* of the rectangle as a function of the width *x* of the rectangle. For what value of *x* if the area largest? What is the maximum area?

22. A farmer with 4000 meters of fencing wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest are that can be enclosed?

23. The half-life of radium is 1690 years. If 10 grams is present now, how much will be present in 50 years?

24. A piece of charcoal is found to contain 30% of the carbon 14 that it originally had. When did the tree from which the charcoal came die? Use 5600 years as the half-life of carbon 14.

25. The population of a southern city follows the exponential law. If the population doubled in size over an 18-month period and the current population is 10,000, what will the population be 2 years from now?

26. Find the amount that results from each investment.

a. $50 invested at 6% compounded monthly after a period of 3 years.
b. $700 invested at 6% compounded daily after a period of 2 years.
c. $100 invested at 12% compounded continuously after a period of $3\frac{3}{4}$ years.

27. Find the principal needed now to get each amount; that is, fond the present value.

a. To get $75 after 3 years at 8% compounded quarterly.
b. To get $300 after 4 years at 3% compounded daily.
c. To get $800 after $2\frac{1}{2}$ years at 8% compounded continuously.