MATH 260 – EXAM #1 **Name:** KEY

Spring Semester 2019

Directions: ***Please show all work for maximum credit.*** *No work = no credit.* Point values for each problem are given. There is a total of 102 points on this exam. This exam will be taken out of 100 points. Please show all work and clearly indicate your answers. Remember, this exam is to show what you know. You may not use any notes, the textbook, mobile phones, or any unauthorized sources for assistance during this exam. You may use a scientific calculator on this exam. Graphing calculators and cell phones are not permitted. Clearly indicate the answer to each question. Any work on separate paper that you would like graded must be indicated on each corresponding problem on this exam. You will need your Mt. SAC student identification card to submit this exam. Good luck!

(10 points) 1. Solve the following system of equations by converting to an augmented matrix and using Gaussian elimination.

$$\begin{aligned} 5x_1 - 4x_2 + \ x_3 &= 15 \\ x_1 - 3x_2 + \ x_3 &= 8 \\ 2x_1 + 4x_2 - 3x_3 &= -4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 5 & -4 & 1 & 15 \\ 1 & -3 & 1 & 8 \\ 2 & 4 & -3 & -4 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 8 \\ 5 & -4 & 1 & 15 \\ 2 & 4 & -3 & -4 \end{array}\right] \xrightarrow[-2R_1+R_3 \to R_3]{-5R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 8 \\ 0 & 11 & -4 & -25 \\ 0 & 10 & -5 & -20 \end{array}\right]$$

$$\xrightarrow{-1R_3+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 8 \\ 0 & 1 & 1 & -5 \\ 0 & 10 & -5 & -20 \end{array}\right] \xrightarrow{-10R_2+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 8 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & -15 & 30 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{15}R_3} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 8 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 8 \\ x_2 + x_3 &= -5 \\ x_3 &= -2 \end{aligned}$$

$x_2 - 2 = -5$

$x_2 = -3$

$x_1 + 9 - 2 = 8$

$x_1 + 7 = 8$

$x_1 = 1$

$(1, -3, -2)$

2. Given the following matrices:

$$A=\begin{bmatrix}7 & 4 & -9\\ 5 & 6 & 2\end{bmatrix},\quad B=\begin{bmatrix}3 & -5 & 4\\ 6 & 8 & -1\end{bmatrix},\quad C=\begin{bmatrix}8 & 4\\ 3 & 7\\ -2 & 5\end{bmatrix},\quad D=\begin{bmatrix}5 & 3\\ 8 & 7\end{bmatrix}$$

Determine the following.

(3 points) a. $-2A+5B$

$$-2\begin{bmatrix}7 & 4 & -9\\ 5 & 6 & 2\end{bmatrix}+5\begin{bmatrix}3 & -5 & 4\\ 6 & 8 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-14 & -8 & 18\\ -10 & -12 & -4\end{bmatrix}+\begin{bmatrix}15 & -25 & 20\\ 30 & 40 & -5\end{bmatrix}=\begin{bmatrix}1 & -33 & 38\\ 20 & 28 & -9\end{bmatrix}$$

(5 points) b. CD

$$\begin{bmatrix}40+32 & 24+28\\ 15+56 & 9+49\\ -10+40 & -6+35\end{bmatrix}=\begin{bmatrix}72 & 52\\ 71 & 58\\ 30 & 29\end{bmatrix}$$

(2 points) c. B^T

$$\begin{bmatrix}3 & 6\\ -5 & 8\\ 4 & -1\end{bmatrix}$$

(4 points) d. D^TC^T

$$\begin{bmatrix}72 & 71 & 30\\ 52 & 58 & 29\end{bmatrix}$$

2 continued. Given the following matrices:

$$A=\begin{bmatrix}7 & 4 & -9\\ 5 & 6 & 2\end{bmatrix},\quad B=\begin{bmatrix}3 & -5 & 4\\ 6 & 8 & -1\end{bmatrix},\quad C=\begin{bmatrix}8 & 4\\ 3 & 7\\ -2 & 5\end{bmatrix},\quad D=\begin{bmatrix}5 & 3\\ 8 & 7\end{bmatrix}$$

(4 points) e. D^2

$$\begin{bmatrix}5 & 3\\ 8 & 7\end{bmatrix}\begin{bmatrix}5 & 3\\ 8 & 7\end{bmatrix}=\begin{bmatrix}25+24 & 15+21\\ 40+56 & 24+49\end{bmatrix}$$

$$=\begin{bmatrix}49 & 36\\ 96 & 73\end{bmatrix}$$

(5 points) f. Solve for the matrix X: $X-3A=2B$

$$X=2B+3A$$

$$X=2\begin{bmatrix}3 & -5 & 4\\ 6 & 8 & -1\end{bmatrix}+3\begin{bmatrix}7 & 4 & -9\\ 5 & 6 & 2\end{bmatrix}$$

$$X=\begin{bmatrix}6 & -10 & 8\\ 12 & 16 & -2\end{bmatrix}+\begin{bmatrix}21 & 12 & -27\\ 15 & 18 & 6\end{bmatrix}$$

$$X=\begin{bmatrix}27 & 2 & -19\\ 27 & 34 & 4\end{bmatrix}$$

(4 points) g. $D(A+B)$

$$\begin{bmatrix}5 & 3\\ 8 & 7\end{bmatrix}\left(\begin{bmatrix}7 & 4 & -9\\ 5 & 6 & 2\end{bmatrix}+\begin{bmatrix}3 & -5 & 4\\ 6 & 8 & -1\end{bmatrix}\right)$$

$$=\begin{bmatrix}5 & 3\\ 8 & 7\end{bmatrix}\begin{bmatrix}10 & -1 & -5\\ 11 & 14 & 1\end{bmatrix}$$

$$=\begin{bmatrix}50+33 & -5+42 & -25+3\\ 80+77 & -8+98 & -40+7\end{bmatrix}$$

$$=\begin{bmatrix}83 & 37 & -22\\ 157 & 90 & -33\end{bmatrix}$$

3. Assume that the following matrices represent an augmented matrix. Determine the solution of the corresponding system of equations. Identify if the system is consistent or inconsistent.

(5 points) a. $\begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 2 & -1 & 2 \end{bmatrix}$

$x_1 + 2x_2 + 3x_3 + x_4 = 1$
$x_2 + 2x_3 - x_4 = 2$
$x_4 = s$
$x_3 = t$
$x_2 = 2 - 2t + s$
$x_1 = 1 - 2(2 - 2t + s) - 3t - s$
$x_1 = -3 + t - 3s$
$\{(t - 3s - 3, 2 - 2t + s, t, s \mid t, s \in \mathbb{R}\}$

consistent

(5 points) b. $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 + 2x_2 = 4$
$x_2 = -1$
$x_1 = 6$
$(6, -1)$ consistent.

(5 points) c. $\begin{bmatrix} 1 & 4 & 2 & 1 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$x_1 + 4x_2 + 2x_3 = 1$
$x_2 + 4x_3 = 3$
$0 = 1 \leftarrow$ inconsistent

(5 points) d. $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 + 2x_2 + x_3 = 2$
$x_2 + 3x_3 = 2$
$x_3 = t$
$x_2 = 2 - 3t$
$x_1 = -2(2 - 3t) - t + 2$
$= 5t - 2$

consistent.

$\{(5t - 2, 2 - 3t, t) \mid t \in \mathbb{R}\}$

(10 points) *4.* Given the following matrix A. Find A^{-1}, if it exists.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 11 \\ 4 & -3 & 10 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 11 & 0 & 1 & 0 \\ 4 & -3 & 10 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-4R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 7 & -2 & 1 & 0 \\ 0 & 1 & 2 & -4 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & -4 & 0 & 1 \\ 0 & 3 & 7 & -2 & 1 & 0 \end{array}\right] \xrightarrow{-3R_2+R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & -4 & 0 & 1 \\ 0 & 0 & 1 & 10 & 1 & -3 \end{array}\right]$$

$$\xrightarrow[-2R_3+R_1 \to R_1]{-2R_3+R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -19 & -2 & 6 \\ 0 & 1 & 0 & -24 & -2 & 7 \\ 0 & 0 & 1 & 10 & 1 & -3 \end{array}\right] \xrightarrow{R_2+R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -43 & -4 & 13 \\ 0 & 1 & 0 & -24 & -2 & 7 \\ 0 & 0 & 1 & 10 & 1 & -3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -43 & -4 & 13 \\ -24 & -2 & 7 \\ 10 & 1 & -3 \end{bmatrix}$$

(4 points) *5.* Evaluate the determinant of the following matrix: $A = \begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$

$30 - 6 = 24$

(1 point each) *6.* Identify the following matrices as either in row-echelon form, reduced row-echelon form, or neither.

a. $\begin{bmatrix} 1 & 5 & 3 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ *b.* $\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ *c.* $\begin{bmatrix} 1 & 4 & 1 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

REF RREF neither

(7 points) *7.* Prove the following statement: Let A be an $n \times n$ invertible matrix. Then, $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$.

$$A^T(A^{-1})^T = (A^{-1}A)^T = I_n^T = I_n$$

$$(A^{-1})^T A^T = (A A^{-1})^T = I_n^T = I_n$$

$$\therefore (A^T)^{-1} = (A^{-1})^T$$

(7 points) *8.* Prove the following statement: Let A and B be $m \times n$ matricies. Then, $A + B = B + A$.

$A = [a_{ij}]$ $B = [b_{ij}]$

$$\begin{aligned} A + B &= [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}] \\ &= [b_{ij} + a_{ij}] \\ &= [b_{ij}] + [a_{ij}] \\ &= B + A \end{aligned}$$

(7 points) *9.* Prove the following statement: Let A and B be $m \times n$ matrices and c be a scalar. Then, $c(A+B) = cA + cB$.

$$c(A+B) = c([a_{ij}]+[b_{ij}]) = c[a_{ij}+b_{ij}]$$
$$= [c(a_{ij}+b_{ij})] = [ca_{ij}+cb_{ij}]$$
$$= [ca_{ij}]+[cb_{ij}]$$
$$= c[a_{ij}] + c[b_{ij}]$$
$$= cA + cB$$

(7 points) *10.* Prove the following statement: Let A, B, and C be invertible $n \times n$ matrices. Then, $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

$$(ABC)(C^{-1}B^{-1}A^{-1}) = AB(CC^{-1})B^{-1}A^{-1}$$
$$= AB\,I)B^{-1}A^{-1} = A(BB^{-1})A = (A\,I)A^{-1} = AA^{-1} = I$$

$$(C^{-1}B^{-1}A^{-1})(ABC) = C^{-1}B^{-1}(A^{-1}A)BC$$
$$= C^{-1}B^{-1}(I\,B)C = C^{-1}(B^{-1}B)C = C^{-1}(IC) = C^{-1}C = I$$

$$\therefore (ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$