MATH 260 – EXAM #1 **Name:** Key
Winter Session 2019

Directions: ***Please show all work for maximum credit.*** *No work = no credit.* Point values for each problem are given. There is a total of 102 points on this exam. This exam will be taken out of 100 points. Please show all work and clearly indicate your answers. Remember, this exam is to show what you know. You may not use any notes, the textbook, mobile phones, or any unauthorized sources for assistance during this exam. You may not use a calculator on this exam. Clearly indicate the answer to each question. Any work on separate paper that you would like graded must be indicated on each corresponding problem on this exam. You will need your Mt. SAC student identification card to submit this exam. Good luck!

(10 points) 7. Solve the following system of equations by converting to an augmented matrix and using Gaussian elimination.

$$\begin{aligned} -2x_1 + 5x_2 - 10x_3 &= 4 \\ x_1 - 2x_2 + 3x_3 &= -1 \\ 7x_1 - 17x_2 + 34x_3 &= -16 \end{aligned}$$

$$\begin{bmatrix} -2 & 5 & -10 & 4 \\ 1 & -2 & 3 & -1 \\ 7 & -17 & 34 & -16 \end{bmatrix} \quad R_1 <-> R_2 \quad \begin{bmatrix} 1 & -2 & 3 & -1 \\ -2 & 5 & -10 & 4 \\ 7 & -17 & 34 & -16 \end{bmatrix}$$

$$2R_1 + R_2 -> R_2 \quad \begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 7 & -17 & 34 & -16 \end{bmatrix} \quad -7R_1 + R_3 -> R_3 \quad \begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 0 & -3 & 13 & -9 \end{bmatrix}$$

$$3R_2 + R_3 -> R_3 \quad \begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$x_1 - 2x_2 + 3x_3 = -1$
$x_2 - 4x_3 = 2$
$x_3 = -3$

$x_2 - 4(-3) = 2$
$x_2 + 12 = 2$
$x_2 = -10$

$x_1 - 2(-10) + 3(-3) = -1$
$x_1 + 20 - 9 = -1$
$x_1 + 11 = -1$
$x_1 = -12$

$(-12, -10, -3)$

2. Given the following matrices:

$$A=\begin{bmatrix}4 & 5 & -2\\ 6 & 8 & 1\end{bmatrix},\quad B=\begin{bmatrix}8 & 3 & -7\\ -2 & 5 & 9\end{bmatrix},\quad C=\begin{bmatrix}4 & 6\\ 3 & -1\\ 5 & 2\end{bmatrix},\quad D=\begin{bmatrix}3 & 4\\ 2 & 1\end{bmatrix}$$

Determine the following.

(3 points) a. $5A-3B$

$$5\begin{bmatrix}4 & 5 & -2\\ 6 & 8 & 1\end{bmatrix}-3\begin{bmatrix}8 & 3 & -7\\ -2 & 5 & 9\end{bmatrix}$$

$$=\begin{bmatrix}20 & 25 & -10\\ 30 & 40 & 5\end{bmatrix}+\begin{bmatrix}-24 & -9 & 21\\ 6 & -15 & -27\end{bmatrix}=\begin{bmatrix}-4 & 16 & 11\\ 36 & 25 & -22\end{bmatrix}$$

(5 points) b. AC

$$\begin{bmatrix}4 & 5 & -2\\ 6 & 8 & 1\end{bmatrix}\begin{bmatrix}4 & 6\\ 3 & -1\\ 5 & 2\end{bmatrix}=\begin{bmatrix}16+15-10 & 24-5-4\\ 24+24+5 & 36-8+2\end{bmatrix}$$

$$=\begin{bmatrix}21 & 15\\ 53 & 30\end{bmatrix}$$

(2 points) c. C^T

$$\begin{bmatrix}4 & 3 & 5\\ 6 & -1 & 2\end{bmatrix}$$

(4 points) d. $C^T A^T$

$$\begin{bmatrix}21 & 53\\ 15 & 30\end{bmatrix}$$

2 continued. Given the following matrices:

$$A=\begin{bmatrix}4&5&-2\\6&8&1\end{bmatrix},\quad B=\begin{bmatrix}8&3&-7\\-2&5&9\end{bmatrix},\quad C=\begin{bmatrix}4&6\\3&-1\\5&2\end{bmatrix},\quad D=\begin{bmatrix}3&4\\2&1\end{bmatrix}$$

(5 points) e. D^2

$$\begin{bmatrix}3&4\\2&1\end{bmatrix}\begin{bmatrix}3&4\\2&1\end{bmatrix}=\begin{bmatrix}9+8&12+4\\6+2&8+1\end{bmatrix}$$

$$=\begin{bmatrix}17&16\\8&9\end{bmatrix}$$

(5 points) f. Solve for the matrix X: $X+2A=3B$

$$X=3B-2A$$

$$X=3\begin{bmatrix}8&3&-7\\-2&5&9\end{bmatrix}-2\begin{bmatrix}4&5&-2\\6&8&1\end{bmatrix}$$

$$=\begin{bmatrix}24&9&-21\\-6&15&27\end{bmatrix}+\begin{bmatrix}-8&-10&4\\-12&-16&-2\end{bmatrix}$$

$$=\begin{bmatrix}16&-1&-17\\-18&-1&25\end{bmatrix}$$

(5 points) g. $D(A+B)$

$$\begin{bmatrix}3&4\\2&1\end{bmatrix}\left(\begin{bmatrix}4&5&-2\\6&8&1\end{bmatrix}+\begin{bmatrix}8&3&-7\\-2&5&9\end{bmatrix}\right)$$

$$=\begin{bmatrix}3&4\\2&1\end{bmatrix}\begin{bmatrix}12&8&-9\\4&13&10\end{bmatrix}$$

$$=\begin{bmatrix}36+16&24+52&-27+40\\24+4&16+13&-18+10\end{bmatrix}=\begin{bmatrix}52&76&13\\28&29&-8\end{bmatrix}$$

3. Assume that the following matrices represent an augmented matrix. Determine the solution of the corresponding system of equations.

(4 points) a. $\begin{bmatrix} 1 & 4 & -2 & 2 \\ 0 & 1 & 5 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$x_1 + 4x_2 - 2x_3 = 2$

$x_2 + 5x_3 = 1$

$x_3 = 1$

$x_2 + 5 = 1$

$x_2 = -4$

$x_1 - 16 - 2 = 2$

$x_1 = 20$

$(20, -4, 1)$

(4 points) b. $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 + 2x_2 + 2x_3 = 1$

$x_2 - 4x_3 = 1$

$x_3 = t$

$x_2 = 4t + 1$

$x_1 + 2(4t+1) + 2t = 1$

$x_1 + 10t + 2 = 1$

$x_1 = -10t - 1$

$(-10t-1, 4t+1, t)$

(4 points) c. $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$x_1 + 3x_2 - x_3 = 2$

$x_2 + 2x_3 = 3$

$0 = 1$

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(4 points) d. $\begin{bmatrix} 1 & 4 & -3 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$x_1 + 4x_2 - 3x_3 = 1$

$x_2 + 2x_3 = 1$

$x_3 = t$

$x_2 = 1 - 2t$

$x_1 + 4(1-2t) - 3t = 1$

$x_1 + 4 - 11t = 1$

$x_1 = 11t - 3$

$(11t-3, -2t+1, t)$

(10 points) *4.* Given the following matrix A. Find A^{-1}, if it exists.

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 7 & -6 \\ 2 & -3 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ -3 & 7 & -6 & 0 & 1 & 0 \\ 2 & -3 & 0 & 0 & 0 & 1 \end{array}\right] \quad 3R_1 + R_2 \to R_2 \quad \begin{bmatrix} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 3 & 1 & 0 \\ 2 & -3 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$-2R_1 + R_3 \to R_3$

$$\begin{bmatrix} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 3 & 1 & 0 \\ 0 & 1 & -2 & -2 & 0 & 1 \end{bmatrix} \quad -1R_2 + R_3 \to R_3 \quad \begin{bmatrix} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 3 & 1 & 0 \\ 0 & 0 & 1 & -5 & -1 & 1 \end{bmatrix}$$

$3R_3 + R_2 \to R_2$

$$\begin{bmatrix} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -12 & -2 & 3 \\ 0 & 0 & 1 & -5 & -1 & 1 \end{bmatrix} \quad -1R_3 + R_1 \to R_1 \quad \begin{bmatrix} 1 & -2 & 0 & 6 & 1 & -1 \\ 0 & 1 & 0 & -12 & -2 & 3 \\ 0 & 0 & 1 & -5 & -1 & 1 \end{bmatrix}$$

$2R_2 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & -18 & -3 & 5 \\ 0 & 1 & 0 & -12 & -2 & 3 \\ 0 & 0 & 1 & -5 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -18 & -3 & 5 \\ -12 & -2 & 3 \\ -5 & -1 & 1 \end{bmatrix}$$

5. Determine the inverse of each of the following elementary matrices.

(3 points) a. $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

$$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}$$

(3 points) b. $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(3 points) c. $E_3 = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$E_3^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(8 points) 6. Determine a polynomial function whose graph passes through the points $(1,4)$, $(2,0)$, and $(3,12)$.

$$y = c + bx + ax^2$$

$(1,4)$: $4 = c + b + a$

$(2,0)$: $0 = c + 2b + 4a$

$(3,12) =$ $12 = c + 3b + 9a$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 0 \\ 1 & 3 & 9 & 12 \end{array}\right] \xrightarrow{-R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & -4 \\ 1 & 3 & 9 & 12 \end{bmatrix} \xrightarrow{-R_1 + R_3 \to R_3} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 8 & 8 \end{bmatrix}$$

$$\xrightarrow{-2R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 2 & 16 \end{bmatrix} \xrightarrow{\frac{1}{2}R_3} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

$c + b + a = 4$
$b + 3a = -4$
$a = 8$

$b + 24 = -4$
$b = -28$

$c - 28 + 8 = 4$
$c - 20 = 4$
$c = 24$

$$y = 8x^2 - 28x + 24$$

(6 points) *7.* Prove the following statement: Let A be an $n\times n$ matrix and c be a nonzero scalar. Then, $(cA)^{-1} = \frac{1}{c}A^{-1}$.

$$(cA)\left(\tfrac{1}{c}A^{-1}\right) = c\cdot\tfrac{1}{c}AA^{-1} = \left(c\cdot\tfrac{1}{c}\right)(AA^{-1}) = 1\cdot I = I$$

$$\left(\tfrac{1}{c}A^{-1}\right)(cA) = \tfrac{1}{c}\cdot cA^{-1}A = \left(\tfrac{1}{c}\cdot c\right)(A^{-1}A) = 1\cdot I = I$$

$$\therefore (cA)^{-1} = \tfrac{1}{c}A^{-1}.$$

(6 points) *8.* Prove the following statement: Let A be an $m\times n$ matrix and let B be an $n\times p$ matrix. Then, $(A+B)^T = A^T + B^T$.

$$(A+B)^T = [a_{ij}+b_{ij}]^T = [a_{ji}+b_{ji}] = [a_{ji}] + [b_{ji}]$$

$$= [a_{ij}]^T + [b_{ij}]$$

$$= A^T + B^T.$$

(6 points) 9. Prove the following statement: Let *A*, *B*, and *C* be $m \times n$ matrices. Then, $A+(B+C)=(A+B)+C$

$$A+(B+C) = [a_{ij}] + [b_{ij}+c_{ij}]$$
$$= [a_{ij} + (b_{ij} + c_{ij})]$$
$$= [(b_{ij} + b_{ij}) + c_{ij}]$$
$$= [a_{ij}+b_{ij}] + [c_{ij}]$$
$$= (A+B)+C$$

(6 points) 10. Prove the following statement: Let *A* and *B* be invertible $n \times n$ matrices. Then, $(AB)^{-1} = B^{-1}A^{-1}$

$$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = A(I)A^{-1} = (AI)A^{-1} = AA^{-1} = I$$

$$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}(I)B = B^{-1}(IB) = B^{-1}B = I$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1}.$$