

(8 pts)

$$\#1 \begin{bmatrix} -1 & 1 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{bmatrix} \xrightarrow{-2R_1 + R_2, -5R_1 + R_3} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 5 & 5 & 0 \\ 0 & 5 & 5 & 0 \end{bmatrix}$$

$$\xrightarrow{-5R_2 + R_3} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{1/5 R_2} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-5R_2 + R_3} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 9 \end{bmatrix}$$

$$\xrightarrow{1/3 R_3} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\begin{aligned} x - y - 2z &= -1 \\ y + z &= 0 \\ z &= 3 \end{aligned}$$

$$\begin{aligned} x + 3 - 6 &= -1 \\ x - 3 &= -1 \\ x &= 2 \end{aligned}$$

(2, -3, 3)

#2)

3 pts
a.) $4A - 5B = 4 \begin{bmatrix} 3 & 2 & -4 \\ 5 & 8 & -1 \end{bmatrix} - 5 \begin{bmatrix} 6 & 5 & 7 \\ 4 & 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 12 & 8 & -16 \\ 20 & 32 & -4 \end{bmatrix} + \begin{bmatrix} -30 & 25 & -35 \\ -20 & -10 & -15 \end{bmatrix} = \begin{bmatrix} -18 & 33 & -51 \\ 0 & 22 & -19 \end{bmatrix}$$

5 pts

b.) $BC = \begin{bmatrix} 6 & -5 & 7 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 9 \\ 5 & -2 \\ 6 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 6 \cdot 3 + (-5) \cdot 5 + 7 \cdot 6 & 6 \cdot 9 + (-5) \cdot (-2) + 7 \cdot 2 \\ 4 \cdot 3 + 2 \cdot 5 + 3 \cdot 6 & 4 \cdot 9 + 2 \cdot (-2) + 3 \cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 18 - 25 + 42 & 54 + 10 + 14 \\ 12 + 10 + 18 & 36 - 4 + 6 \end{bmatrix} = \begin{bmatrix} 35 & 78 \\ 40 & 38 \end{bmatrix}$$

3 pts
c.)

$$CT = \begin{bmatrix} 3 & 5 & 6 \\ 9 & -2 & 2 \end{bmatrix}$$

5pts
d.) $C^T B^T = (BC)^T = \begin{bmatrix} 35 & 40 \\ 78 & 38 \end{bmatrix}$ (from part b)

5pts
e.) $D^2 = \begin{bmatrix} 8 & 4 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 8 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 8 \cdot 8 + 4 \cdot 5 & 8 \cdot 4 + 4 \cdot 3 \\ 5 \cdot 8 + 3 \cdot 5 & 5 \cdot 4 + 3 \cdot 3 \end{bmatrix}$
 $= \begin{bmatrix} 64 + 20 & 32 + 12 \\ 40 + 15 & 20 + 9 \end{bmatrix} = \begin{bmatrix} 84 & 44 \\ 55 & 29 \end{bmatrix}$

5pts
f.) $D(A+B) = \begin{bmatrix} 8 & 4 \\ 5 & 3 \end{bmatrix} \left(\begin{bmatrix} 3 & 2 & -4 \\ 5 & 8 & -1 \end{bmatrix} + \begin{bmatrix} 6 & -5 & 7 \\ 4 & 2 & 3 \end{bmatrix} \right)$
 $= \begin{bmatrix} 8 & 4 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 9 & -3 & 3 \\ 9 & 10 & 2 \end{bmatrix} = \begin{bmatrix} 8 \cdot 9 + 4 \cdot 9 & 8 \cdot (-3) + 4 \cdot 10 & 8 \cdot 3 + 4 \cdot 2 \\ 5 \cdot 9 + 3 \cdot 9 & 5 \cdot (-3) + 3 \cdot 10 & 5 \cdot 3 + 3 \cdot 2 \end{bmatrix}$
 $= \begin{bmatrix} 72 + 36 & -24 + 40 & 24 + 8 \\ 45 + 27 & -15 + 30 & 15 + 6 \end{bmatrix} = \begin{bmatrix} 108 & 16 & 32 \\ 72 & 15 & 21 \end{bmatrix}$

#3) (3pts)
9.) $\begin{bmatrix} 1 & 3 & -5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 + 3x_2 - 5x_3 = 2 \\ x_2 + 4x_3 = 3 \\ 0 = 0 \end{cases}$

$$\begin{aligned} x_3 &= t \\ x_2 &= 3 - 4t \\ x_1 &= 2 - 3(3 - 4t) + 5t \\ x_1 &= 2 - 9 + 12t + 5t \\ x_1 &= -7 + 17t \end{aligned}$$

$$(-7 + 17t, 3 - 4t, t)$$

(3pts)
b.) $\begin{bmatrix} 1 & 2 & 5 & 1 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Rightarrow \begin{cases} x_1 + 2x_2 + 5x_3 = 1 \\ x_2 + 4x_3 = -3 \\ x_3 = 5 \end{cases}$

$$\begin{aligned} x_2 &= -3 - 20 = -23 \\ x_1 &= 1 + 46 - 25 \\ &= 22 \end{aligned}$$

$$(22, -23, 5)$$

(3pts)
c.) $\begin{bmatrix} 1 & -3 & 2 & 5 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{cases} x_1 - 3x_2 + 2x_3 = 5 \\ x_2 + 3x_3 = -4 \\ 0 = 1 \end{cases}$

No solution

(3 pts)

$$d.) \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 4 \end{bmatrix}$$

$$x_1 + 2x_2 + 4x_3 = 3$$

$$x_2 + 2x_3 = 4$$

$$x_3 = t \quad x_2 = 4 - 2t$$

$$(-5, 4 - 2t, t)$$

$$x_1 = 3 - 2(4 - 2t) - 4t$$

$$x_1 = 3 - 8 + 4t - 4t$$

$$x_1 = -5$$

(8 pts)

#4.)

$$\begin{bmatrix} 2 & 3 & 1 & 1 & 0 & 0 \\ 2 & -3 & -3 & 0 & 1 & 0 \\ 4 & 0 & 3 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 2 & -3 & -3 & 0 & 1 & 0 \\ 4 & 0 & 3 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{-2R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & -6 & -4 & -1 & 1 & 0 \\ -4 & 0 & 3 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-4R_1 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & -6 & -4 & -1 & 1 & 0 \\ 0 & -6 & 1 & -2 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{6}R_2} \begin{bmatrix} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 2/3 & 1/6 & -1/6 & 0 \\ 0 & -6 & 1 & -2 & 0 & 1 \\ 0 & 6 & 4 & 1 & -1 & 0 \end{bmatrix} \xrightarrow{-6R_2 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 2/3 & 1/6 & -1/6 & 0 \\ 0 & 0 & 5 & -1 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{5}R_3} \begin{bmatrix} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 2/3 & 1/6 & -1/6 & 0 \\ 0 & 0 & 1 & -1/5 & -1/5 & 1/5 \\ 0 & 0 & -2/3 & 2/15 & 2/15 & -2/15 \end{bmatrix} \xrightarrow{-2/3R_3 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 3/10 & -1/30 & -2/15 \\ 0 & 0 & 1 & -1/5 & -1/5 & 1/5 \\ 0 & 0 & -1/2 & 1/10 & 1/10 & -1/10 \end{bmatrix}$$

$$\xrightarrow{-1/2R_3 + R_4 \rightarrow R_4} \begin{bmatrix} 1 & 3/2 & 0 & 3/5 & 1/10 & -1/10 \\ 0 & 1 & 0 & 3/10 & -1/30 & -2/15 \\ 0 & 0 & 1 & -1/5 & -1/5 & 1/5 \\ 0 & -3/2 & 0 & -9/20 & 1/20 & -1/5 \end{bmatrix} \xrightarrow{-3/2R_2 + R_1 \rightarrow R_1} \begin{bmatrix} 1 & 0 & 0 & 2/20 & 3/20 & 1/10 \\ 0 & 1 & 0 & 3/10 & -1/30 & -2/15 \\ 0 & 0 & 1 & -1/5 & -1/5 & 1/5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3/20 & 3/20 & 1/10 \\ 3/10 & -1/30 & -2/15 \\ -1/5 & -1/5 & 1/5 \end{bmatrix}$$

(8 pts)
#5

$$f(x) = a_0 + a_1x + a_2x^2$$

$$\begin{aligned} 5 &= a_0 + a_1 \cdot 2 + a_2(2)^2 \\ 0 &= a_0 + a_1 \cdot 3 + a_2(3)^2 \\ 20 &= a_0 + a_1 \cdot 4 + a_2(4)^2 \end{aligned}$$

$$\begin{aligned} a_0 + 2a_1 + 4a_2 &= 5 \\ a_0 + 3a_1 + 9a_2 &= 0 \\ a_0 + 4a_1 + 16a_2 &= 20 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 1 & 3 & 9 & 0 \\ 1 & 4 & 16 & 20 \end{bmatrix} \xrightarrow{-R_1+R_2 \rightarrow R_2} \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -5 \\ 1 & 4 & 16 & 20 \end{bmatrix} \xrightarrow{-R_1+R_3 \rightarrow R_3} \begin{bmatrix} 0 & -2 & -10 & 10 \\ 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -5 \end{bmatrix}$$

$$\xrightarrow{-2R_2+R_3 \rightarrow R_3} \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -5 \\ 0 & 0 & 2 & 25 \end{bmatrix} \xrightarrow{\frac{1}{2}R_3} \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -5 \\ 0 & 0 & 1 & \frac{25}{2} \end{bmatrix}$$

$$\begin{aligned} a_0 + 2a_1 + 4a_2 &= 5 \\ a_1 + 5a_2 &= 5 \\ a_2 &= \frac{25}{2} \end{aligned}$$

$$\begin{aligned} a_1 + 5\left(\frac{25}{2}\right) &= 5 \\ a_1 &= 5 - \frac{125}{2} \\ a_1 &= -\frac{135}{2} \end{aligned}$$

$$\begin{aligned} a_0 - 135 + 50 &= 5 \\ a_0 - 85 &= 5 \\ a_0 &= 90 \end{aligned}$$

$$f(x) = 90 - \frac{135}{2}x + \frac{25}{2}x^2$$

(8 pts)
#6

$$\begin{aligned} 500 - 300 &= x_1 + x_2 \\ x_1 + x_3 &= x_4 + 150 \\ x_2 + 200 &= x_3 + x_5 \\ x_4 + x_5 &= 350 \end{aligned}$$

$$\begin{aligned} x_1 + x_2 &= 300 \\ x_1 + x_3 - x_4 &= 150 \\ -x_2 + x_3 + x_5 &= 200 \\ x_4 + x_5 &= 350 \end{aligned}$$

$$\begin{bmatrix} -1 & -1 & 0 & 0 & 0 & -300 \\ 1 & 1 & 0 & 0 & 0 & 300 \\ 1 & 0 & 1 & -1 & 0 & 150 \\ 0 & -1 & 1 & 0 & 1 & 200 \\ 0 & 0 & 0 & 1 & 1 & 350 \end{bmatrix} \xrightarrow{-R_1+R_2 \rightarrow R_2} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 300 \\ 0 & -1 & 1 & -1 & 0 & -150 \\ 0 & -1 & 1 & 0 & 1 & 200 \\ 0 & 0 & 0 & 1 & 1 & 350 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 300 \\ 0 & 1 & -1 & 1 & 0 & 150 \\ 0 & -1 & 1 & 0 & 1 & 200 \\ 0 & 0 & 0 & 1 & 1 & 350 \end{bmatrix}$$

$$\xrightarrow{R_2+R_3 \rightarrow R_3} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 300 \\ 0 & 1 & -1 & 1 & 0 & 150 \\ 0 & 0 & 0 & 1 & 1 & 350 \\ 0 & 0 & 0 & 1 & 1 & 350 \end{bmatrix} \xrightarrow{-R_3+R_4 \rightarrow R_4} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 300 \\ 0 & 1 & -1 & 1 & 0 & 150 \\ 0 & 0 & 0 & 1 & 1 & 350 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 + x_2 &= 300 & x_5 &= t \\ x_2 - x_3 + x_4 &= 150 & x_4 &= 350 - t \\ x_4 + x_5 &= 350 & x_3 &= 5 \\ x_2 &= 150 + 5 - (350 - t) \\ &= -200 + 5 + t \\ x_1 &= 300 - (-200 + 5 + t) \\ &= 500 - 5 - t \end{aligned}$$

$$(500 - 5 - t, -200 + 5 + t, 5, 350 - t, t)$$

(5pts)
#7

$$(cA)^T = (c[a_{ij}])^T = (c a_{ij})^T = [c a_{ji}] = c [a_{ji}] = c [a_{ij}]^T = c A^T$$

(5pts)

#8)

$$\begin{aligned}(c+d)A &= (c+d)[a_{ij}] = [(c+d)a_{ij}] = [ca_{ij} + da_{ij}] = [ca_{ij}] + [da_{ij}] \\ &= c[a_{ij}] + d[a_{ij}] = cA + dA.\end{aligned}$$