MATH 260 Exam #2 KEY

#1)

(2 pts) a.)

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Inverse} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2 pts) b.)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix} \quad \text{Inverse} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(2 pts) c.)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Inverse} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(8 pts) #2.)

$$\begin{vmatrix} 4 & -7 & 9 & 1 \\ 6 & 2 & 7 & 0 \\ 3 & 6 & -3 & 3 \\ 0 & 7 & 4 & -1 \end{vmatrix} \overset{\frac{1}{3}R_3}{=} 3 \begin{vmatrix} 4 & -7 & 9 & 1 \\ 6 & 2 & 7 & 0 \\ 1 & 2 & -1 & 1 \\ 0 & 7 & 4 & -1 \end{vmatrix} \overset{R_1 \Leftrightarrow R_3}{=} -3 \begin{vmatrix} 1 & 2 & -1 & 1 \\ 6 & 2 & 7 & 0 \\ 4 & -7 & 9 & 1 \\ 0 & 7 & 4 & -1 \end{vmatrix} \overset{-6R_1+R_2}{=} -3 \begin{vmatrix} 1 & 2 & -1 & 1 \\ 0 & -10 & 13 & -6 \\ 4 & -7 & 9 & 1 \\ 0 & 7 & 4 & -1 \end{vmatrix}$$

-6 -12 6 -6

-4 -8 4 -4

$$\overset{-4R_1+R_3}{=} -3 \begin{vmatrix} 1 & 2 & -1 & 1 \\ 0 & -10 & 13 & -6 \\ 0 & -15 & 13 & -3 \\ 0 & 7 & 4 & -1 \end{vmatrix} \overset{-\frac{1}{10}R_2}{=} -3(-10) \begin{vmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{13}{10} & \frac{3}{5} \\ 0 & -15 & 13 & -3 \\ 0 & 7 & 4 & -1 \end{vmatrix} \overset{15R_2+R_3}{=} 30 \begin{vmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{13}{10} & \frac{3}{5} \\ 0 & 0 & -\frac{13}{2} & 6 \\ 0 & 7 & 4 & -1 \end{vmatrix} \overset{-7R_2+R_4}{=} 30 \begin{vmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{13}{10} & \frac{3}{5} \\ 0 & 0 & -\frac{13}{2} & 6 \\ 0 & 0 & \frac{131}{10} & -\frac{26}{5} \end{vmatrix}$$

0 15 $-\frac{39}{2}$ 9

0 -7 $\frac{91}{10}$ $-\frac{21}{5}$

0 0 $-\frac{131}{10}$ $\frac{786}{65}$

$$\overset{-\frac{2}{13}R_3}{=} 30\left(-\frac{13}{2}\right) \begin{vmatrix} 1 & -2 & -1 & 1 \\ 0 & 1 & -\frac{13}{10} & \frac{3}{5} \\ 0 & 0 & 1 & -\frac{12}{13} \\ 0 & 0 & \frac{131}{10} & -\frac{26}{5} \end{vmatrix} \overset{-\frac{131}{10}R_3+R_4}{=} -195 \begin{vmatrix} 1 & -2 & -1 & 1 \\ 0 & 1 & -\frac{13}{10} & \frac{3}{5} \\ 0 & 0 & 1 & -\frac{12}{13} \\ 0 & 0 & 0 & \frac{448}{65} \end{vmatrix}$$

$$\det(A) = (-195)(1)(1)(1)\left(\frac{448}{65}\right)$$

$$= -1344$$

(6pts)
#3

$$\rightarrow \begin{vmatrix} 3 & 8 & -7 \\ 0 & -5 & 4 \\ 8 & 1 & 6 \end{vmatrix} = 0 + (-5)\begin{vmatrix} 3 & -7 \\ 8 & 6 \end{vmatrix} - (4)\begin{vmatrix} 3 & 8 \\ 8 & 1 \end{vmatrix}$$

$$= -5(18+56) - 4(3-64)$$

$$= -5(74) - 4(-61)$$

$$= -370 + 244 = -126$$

#4) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$

(6pts)
a.)

$C_{11} = \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} = 4$ $\quad C_{12} = (-1)\begin{vmatrix} 0 & -1 \\ 2 & 2 \end{vmatrix} = 2$ $\quad C_{13} = \begin{vmatrix} 0 & 1 \\ 2 & 2 \end{vmatrix} = -2$

$C_{21} = (-1)\begin{vmatrix} 2 & 3 \\ 2 & 2 \end{vmatrix} = -2$ $\quad C_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = -4$ $\quad C_{23} = (-1)\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = -2$

$C_{31} = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -5$ $\quad C_{32} = (-1)\begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} = 1$ $\quad C_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$

$M_c = \begin{bmatrix} 4 & 2 & -2 \\ -2 & -4 & -2 \\ -5 & -1 & 1 \end{bmatrix}$ $\qquad M_c^T = \text{adj}(A) = \begin{bmatrix} 4 & -2 & -5 \\ 2 & -4 & -1 \\ -2 & -2 & 1 \end{bmatrix}$

(3pts)
b.) $\det(A) = 0 + 1\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 1(-4) - (-1)(-2)$

$$= -4 - 2 = -6$$

(2pts)
#5.) $18x_1 + 12x_2 = 13$

$30x_1 + 24x_2 = 23$

$$A = \begin{vmatrix} 18 & 12 \\ 30 & 24 \end{vmatrix} = 432 - 360 = 72$$

$$x_1 = \frac{\begin{vmatrix} 13 & 12 \\ 23 & 24 \end{vmatrix}}{72} = \frac{36}{72} = \frac{1}{2} \qquad x_2 = \frac{\begin{vmatrix} 18 & 13 \\ 30 & 23 \end{vmatrix}}{72} = \frac{24}{72} = \frac{1}{3} \qquad \left(\tfrac{1}{2}, \tfrac{1}{3}\right)$$

#6) $\vec{u} = (3,5,-2)$ $\vec{v} = (7,-4,3)$

(2pts)
a.) $4\vec{u} - 5\vec{v} = 4(3,5,-2) - 5(7,-4,3) = (12,20,-8) + (-35,20,-15) = (-23,40,-23)$

(2pts)
b.) $2(3\vec{u} + 4\vec{v}) = 6\vec{u} + 8\vec{v} = 6(3,5,-2) + 8(7,-4,3) = (18,30,-12) + (56,-32,24) = (74,-2,12)$

#7) $\det(A) = 4$ $\det(B) = 2$ 3x3 matrices

(2pts)
a.) $\det(AB) = 4 \cdot 2 = 8$

(2pts)
b.) $\det(3B) = 3^3(2) = 27 \cdot 2 = 54$

(2pts)
c.) $\det(A^T) = 4$

(2pts)
d.) $\det(B^{-1}) = \frac{1}{\det(B)} = \frac{1}{2}$

(2pts)
e.) $\det(A^2) = (\det(A))^2 = 4^2 = 16$

(2pts)
f.) $\det(C) = -2$

(2pts)
g.) $\det(D) = 4$

(2pts)
h.) $\det(F) = 4 \cdot 2 = 8$

#8)

(4pts)
a.) $(1, b_1, c_1) + (1, b_2, c_2) = (2, b_1+b_2, c_1+c_2) \notin V$

So, V is not closed under addition

(4pts)
b.) $\frac{1}{2} \cdot \pi = \frac{\pi}{2} \notin \mathbb{Q}$ So, $\mathbb{Q}$ is not closed under scalar multiplication

(4pts)
c.) $x^3 - x^3 = 0 \notin V$ So, V is not closed under addition

(5pts)
#10) $(c+d)\vec{u} = (c+d)(u_1, u_2, u_3) = ((c+d)u_1, (c+d)u_2, (c+d)u_3) = (cu_1+du_1, cu_2+du_2, cu_3+du_3)$

$= (cu_1, cu_2, cu_3) + (du_1, du_2, du_3) = c(u_1, u_2, u_3) + d(u_1, u_2, u_3) = c\vec{u} + d\vec{u}$

(5pts)
#11)

$$A+B = \begin{bmatrix} a_1 & 0 \\ b_1 & c_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ b_2 & c_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & 0 \\ b_1+b_2 & c_1+c_2 \end{bmatrix} = \begin{bmatrix} a_2+a_1 & 0 \\ b_2+b_1 & c_2+c_1 \end{bmatrix}$$

$$= \begin{bmatrix} a_2 & 0 \\ b_2 & c_2 \end{bmatrix} + \begin{bmatrix} a_1 & 0 \\ b_1 & c_1 \end{bmatrix} = B + A$$