Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(6 points) 1. Solve the following system of equations by using Gaussian elimination.

$$4x_1 - 3x_2 + 6x_3 = 2$$
$$x_1 - 3x_2 + 6x_3 = 5$$
$$-2x_1 + 3x_2 - 8x_3 = -6$$

$$\begin{bmatrix} 4 & -3 & 6 & | & 2 \\ 1 & -3 & 6 & | & 5 \\ -2 & 3 & -8 & | & -6 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 1 & -3 & 6 & | & 5 \\ 4 & -3 & 6 & | & 2 \\ -2 & 3 & -8 & | & -6 \end{bmatrix} \quad -4R_1 + R_2 \rightarrow R_2 \quad \begin{bmatrix} 1 & -3 & 6 & | & 5 \\ 0 & 9 & -18 & | & -18 \\ -2 & 3 & -8 & | & -6 \end{bmatrix}$$

$$2R_1 + R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & -3 & 6 & | & 5 \\ 0 & 9 & -18 & | & -18 \\ 0 & -3 & -4 & | & -4 \end{bmatrix} \quad \tfrac{1}{9} R_2 \quad \begin{bmatrix} 1 & -3 & 6 & | & 5 \\ 0 & 1 & -2 & | & -2 \\ 0 & -3 & -4 & | & -4 \end{bmatrix} \quad 3R_2 + R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & -3 & 6 & | & 5 \\ 0 & 1 & -2 & | & -2 \\ 0 & 0 & -10 & | & -10 \end{bmatrix} \quad -\tfrac{1}{10} R_3 \quad \begin{bmatrix} 1 & -3 & 6 & | & 5 \\ 0 & 1 & -2 & | & -2 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \Rightarrow$$

$$x_1 - 3x_2 + 6x_3 = 5$$
$$x_2 - 2x_3 = -2$$
$$x_3 = 1$$

$$x_2 - 2(1) = -2$$
$$x_2 = 0$$

$$x_1 - 3(0) + 6(1) = 5$$
$$x_1 = -1$$

$$(-1, 0, 1)$$

(2 points) 2. Given the following augmented matrix: $\begin{bmatrix} 1 & 2 & 1 & | & 3 \\ 0 & 1 & 2 & | & 2 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

Determine the solution to the corresponding system. If the system has an infinite number of solutions, write the solution parametrically.

$$x_1 + 2x_2 + x_3 = 3$$
$$x_2 + 2x_3 = 2 \qquad x_2 = 2 - 2t$$
$$x_3 = t$$

$$(3t - 1, 2 - 2t, t)$$

$$x_1 + 2(2 - 2t) + t = 3 \qquad x_1 + 4 - 3t = 3$$
$$x_1 + 4 - 4t + t = 3 \qquad x_1 = 3t - 1$$

3. Determine if the following matrices are in row-echelon form, reduced row-echelon form, or neither.

(1 point) a. $\begin{bmatrix} 1 & 3 & 4 & 5 & 2 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ *Row-echelon form*

(1 point) b. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 1 & 1 \end{bmatrix}$ *Neither*

(1 point) c. $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ *Reduced Row-echelon form*

4. Given the following matrices: $A = \begin{bmatrix} 5 & -1 & 3 & 7 \\ 2 & 8 & -4 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 6 & -1 & -7 \\ 3 & 0 & 4 & 9 \end{bmatrix}$

Determine the following:

(2 points) a. $A + B$ $\begin{bmatrix} 3 & 5 & 2 & 0 \\ 5 & 8 & 0 & 7 \end{bmatrix}$

(2 points) b. $3A$ $\begin{bmatrix} 15 & -3 & 9 & 21 \\ 6 & 24 & -12 & -6 \end{bmatrix}$

(2 points) c. $A - B$ $\begin{bmatrix} 7 & -7 & 4 & 14 \\ -1 & 8 & -8 & -11 \end{bmatrix}$