

7pts  
#1)

$$\begin{bmatrix} 4 & -3 & 6 & 2 \\ 1 & -3 & 6 & 5 \\ -2 & 3 & 8 & -6 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -3 & 6 & 5 \\ 4 & -3 & 6 & 2 \\ -2 & 3 & 8 & -6 \end{bmatrix} \xrightarrow{-4R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & -3 & 6 & 5 \\ 0 & 9 & -18 & -18 \\ -2 & 3 & 8 & -6 \end{bmatrix}$$

$$\xrightarrow{2R_1 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & -3 & 6 & 5 \\ 0 & 9 & -18 & -18 \\ 0 & -3 & 4 & 4 \end{bmatrix} \xrightarrow{\frac{1}{9}R_2} \begin{bmatrix} 1 & -3 & 6 & 5 \\ 0 & 1 & -2 & -2 \\ 0 & -3 & 4 & 4 \end{bmatrix} \xrightarrow{3R_2 + R_3} \begin{bmatrix} 1 & -3 & 6 & 5 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{2}R_3} \begin{bmatrix} 1 & -3 & 6 & 5 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} x_1 - 3x_2 + 6x_3 &= 5 \\ x_2 - 2x_3 &= -2 \\ x_3 &= 1 \end{aligned}$$

$$\begin{aligned} x_1 + 0 + 6 &= 5 \\ x_2 - 2 &= -2 \\ x_2 &= 0 \end{aligned}$$

$$x_1 = -1$$

$$x_2 = 0$$

7pts

$$\begin{bmatrix} 2 & 1 & -1 & 2 & -6 \\ 2 & 4 & 0 & 1 & 1 \\ 1 & 5 & 2 & 6 & -3 \\ 5 & 2 & -1 & -1 & 3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 2 & 4 & 0 & 1 & 1 \\ 2 & 1 & -1 & 2 & -6 \\ 5 & 2 & -1 & -1 & 3 \end{bmatrix} \xrightarrow{-2R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & -6 & -4 & -11 & 7 \\ 2 & 1 & -1 & 2 & -6 \\ 5 & 2 & -1 & -1 & 3 \end{bmatrix}$$

$$\xrightarrow{-2R_1 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & -6 & -4 & -11 & 7 \\ 0 & -9 & -5 & -10 & 0 \\ 5 & 2 & -1 & -1 & 3 \end{bmatrix}$$

$$\xrightarrow{-5R_1 + R_4 \rightarrow R_4} \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & -6 & -4 & -11 & 7 \\ 0 & -9 & -5 & -10 & 0 \\ 0 & -23 & -11 & -31 & 18 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{6}R_2} \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{2}{3} & \frac{11}{6} & -\frac{7}{6} \\ 0 & -9 & -5 & -10 & 0 \\ 0 & -23 & -11 & -31 & 18 \end{bmatrix}$$

$$\xrightarrow{9R_2 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{2}{3} & \frac{11}{6} & -\frac{7}{6} \\ 0 & 0 & 1 & \frac{13}{2} & -\frac{21}{2} \\ 0 & -23 & -11 & -31 & 18 \end{bmatrix}$$

$$\xrightarrow{23R_2 + R_4 \rightarrow R_4} \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{2}{3} & \frac{11}{6} & -\frac{7}{6} \\ 0 & 0 & 1 & \frac{13}{2} & -\frac{21}{2} \\ 0 & 0 & \frac{13}{3} & \frac{67}{6} & -\frac{53}{6} \end{bmatrix}$$

$$\xrightarrow{-13R_3 + R_4 \rightarrow R_4} \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{2}{3} & \frac{11}{6} & -\frac{7}{6} \\ 0 & 0 & 1 & \frac{13}{2} & -\frac{21}{2} \\ 0 & 0 & 0 & -\frac{5}{3} & \frac{11}{3} \end{bmatrix}$$

$$\xrightarrow{\frac{3}{51}R_4} \begin{bmatrix} 1 & 5 & 2 & 6 & -3 \\ 0 & 1 & \frac{2}{3} & \frac{11}{6} & -\frac{7}{6} \\ 0 & 0 & 1 & \frac{13}{2} & -\frac{21}{2} \\ 0 & 0 & 0 & 1 & -\frac{110}{51} \end{bmatrix}$$

$$\begin{aligned} x_1 + 5x_2 + 2x_3 + 6x_4 &= -3 \\ x_2 + \frac{2}{3}x_3 + \frac{11}{6}x_4 &= -\frac{7}{6} \\ x_3 + \frac{13}{2}x_4 &= -\frac{21}{2} \\ x_4 &= -\frac{110}{51} \end{aligned}$$

$$\begin{aligned} x_3 - \frac{715}{51} &= -\frac{21}{2} \\ x_3 &= \frac{359}{102} \end{aligned}$$

$$x_2 + \frac{2}{3}\left(\frac{359}{102}\right) + \frac{11}{6}\left(-\frac{110}{51}\right) = -\frac{7}{6}$$

$$x_2 + \frac{359}{153} + \left(-\frac{605}{153}\right) = -\frac{7}{6}$$

$$x_2 - \frac{246}{153} = -\frac{7}{6}$$

$$\left(\frac{71}{102}, \frac{15}{34}, \frac{359}{102}, -\frac{110}{51}\right)$$

$$x_1 + 5x_2 + 2x_3 + 6x_4 = -3$$

$$x_1 + 5\left(\frac{15}{34}\right) + 2\left(\frac{359}{102}\right) + 6\left(-\frac{110}{51}\right) = -3$$

$$x_1 = -3 + \frac{377}{102}$$

$$x_1 = \frac{71}{102}$$

$$x_2 = -\frac{7}{6} + \frac{246}{153}$$

$$x_2 = \frac{135}{306}$$

$$x_2 = \frac{15}{34}$$

(3pts)

#3a)

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 + x_3 = 3$$

$$x_2 + x_3 = 2$$

$$x_3 = t$$

$$x_2 = 2 - 2t$$

$$x_1 + 2(2 - 2t) + t = 3$$

$$x_1 + 4 - 4t + t = 3$$

$$x_1 = -1 + 3t$$

$$(-1 + 3t, 2 - 2t, t)$$

(3pts)

#3b)

$$\begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x_1 + 3x_2 + 5x_3 = -2$$

$$x_2 + 4x_3 = 7$$

$$0 = 1$$

No solution