MATH 260 – QUIZ #2 Name: Key

Directions: This is a take home quiz. This quiz is due at the beginning of class on Monday, January 28, 2019. Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(4 points) *1.* Use cofactors to find det(A).

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 4 \\ 2 & -1 & 3 \end{bmatrix}$$

Using 1st column:

$$(-1)(-1)^{1+1}\begin{vmatrix} 1 & 4 \\ -1 & 3 \end{vmatrix} + 0(-1)^{1+2}\begin{vmatrix} 2 & 3 \\ -1 & 3 \end{vmatrix} + 2(-1)^{1+3}\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$$

$$= -1(3+4)+2(8-3)$$

$$= -7+10=3$$

2. Given the following matrix: $A = \begin{bmatrix} -2 & 3 & -1 \\ 2 & 1 & 5 \\ 0 & 2 & 3 \end{bmatrix}$

(3 points) *a.* Find adj(A).

$$M_c = \begin{bmatrix} -7 & -6 & 4 \\ -11 & -6 & 4 \\ 16 & 8 & -8 \end{bmatrix}$$

$$\text{adj}(A) = M_c^T = \begin{bmatrix} -7 & -11 & 16 \\ -6 & -6 & 8 \\ 4 & 4 & -8 \end{bmatrix}$$

(2 points) *b.* Use the result from part *a* to find A^{-1}.

$$\det(A) = -2(-7)+2(-11) = 14-22=-8$$

$$A^{-1} = \frac{1}{\det(A)}\text{adj}(A) = -\frac{1}{8}\begin{bmatrix} -7 & -11 & 16 \\ -6 & -6 & 8 \\ 4 & 4 & -8 \end{bmatrix}$$

(4 points) 3. Use elementary row operations to rewrite the following matrix as an upper triangular matrix or lower triangular matrix in order to find det(A).

$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 4 & 3 \\ 5 & 2 & 5 & 3 \end{bmatrix}$$

$$\begin{vmatrix} 2 & 1 & 3 & 5 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 4 & 3 \\ 5 & 2 & 5 & 3 \end{vmatrix} \overset{-R_1+R_2 \to R_2}{=} \begin{vmatrix} 2 & 1 & 3 & 5 \\ 1 & -1 & -2 & -3 \\ 4 & 1 & 4 & 3 \\ 5 & 2 & 5 & 3 \end{vmatrix} \overset{R_1 \leftrightarrow R_2}{=} - \begin{vmatrix} 1 & -1 & -2 & -3 \\ 2 & 1 & 3 & 5 \\ 4 & 1 & 4 & 3 \\ 5 & 2 & 5 & 3 \end{vmatrix} \overset{\substack{-2R_1+R_2 \to R_2 \\ -4R_1+R_3 \to R_3 \\ -5R_1+R_4 \to R_4}}{=} - \begin{vmatrix} 1 & -1 & -2 & -3 \\ 0 & 3 & 7 & 11 \\ 0 & 5 & 12 & 15 \\ 0 & 7 & 15 & 18 \end{vmatrix}$$

$$\overset{-R_3+R_4 \to R_4}{=} - \begin{vmatrix} 1 & -1 & -2 & -3 \\ 0 & 3 & 7 & 11 \\ 0 & 5 & 12 & 15 \\ 0 & 2 & 3 & 3 \end{vmatrix} \overset{-R_4+R_2 \to R_2}{=} - \begin{vmatrix} 1 & -1 & -2 & -3 \\ 0 & 1 & 4 & 8 \\ 0 & 5 & 12 & 15 \\ 0 & 2 & 3 & 3 \end{vmatrix} \overset{\substack{-5R_2+R_3 \to R_3 \\ -2R_2+R_4 \to R_4}}{=} - \begin{vmatrix} 1 & -1 & -2 & -3 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & -8 & -25 \\ 0 & 0 & -5 & -13 \end{vmatrix}$$

$$\overset{\substack{-2R_3 \\ 3R_4}}{=} \frac{1}{6} \begin{vmatrix} 1 & -1 & -2 & -3 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & 16 & 50 \\ 0 & 0 & -15 & -39 \end{vmatrix} \overset{R_4+R_3 \to R_3}{=} \frac{1}{6} \begin{vmatrix} 1 & -1 & -2 & -3 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & 1 & 11 \\ 0 & 0 & -15 & -39 \end{vmatrix} \overset{15R_3+R_4 \to R_4}{=} \frac{1}{6} \begin{vmatrix} 1 & -1 & -2 & -3 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & 1 & 11 \\ 0 & 0 & 0 & 126 \end{vmatrix} = \frac{1}{6} \cdot 126 = 21$$

4. Given that A and B are 3x3 matrices and that $\det(A) = 6$ and $\det(B) = 4$.

(1 point) a. Find $\det(3B)$. $= 3^3 \cdot 4 = 27 \cdot 4 = 108$

(1 point) b. Find $\det(AB)$. $= 6 \cdot 4 = 24$

(1 point) c. Find $\det(B^T)$. $= 4$

(1 point) d. Find $\det(A^{-1})$. $= \frac{1}{6}$

(1 point) e. If matrix C is the result of performing $R_1 \leftrightarrow R_2$ to matrix A, find $\det(C)$. $\det C = -6$

(1 point) f. If matrix D is the result of performing $3R_1 + R_3 \to R_3$ to matrix B, find $\det(D)$. $\det D = 4$