MATH 260 – QUIZ #3 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(4 points) 1. Use Cramer's Rule to solve the following system of linear equations.

$$2x_1 - 3x_2 = 2$$
$$x_1 + 2x_2 = 4$$

$$\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 4+3=7$$

$$x_1 = \frac{\begin{vmatrix} 2 & -3 \\ 4 & 2 \end{vmatrix}}{7} = \frac{4+12}{7} = \frac{16}{7}$$

$$x_2 = \frac{\begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix}}{7} = \frac{8-2}{7} = \frac{6}{7}$$

$$\left(\frac{16}{7}, \frac{6}{7}\right)$$

2. Given the following vectors: $\vec{u} = (5,3,-2)$, $\vec{v} = (2,-4,1)$

Evaluate the following.

(2 points) a. $5\vec{u} - 3\vec{v}$

$= 5(5,3,-2) - 3(2,-4,1)$

$= (25,15,-10) + (-6,12,-3) = (19,27,-13)$

(2 points) b. $3(\vec{u} + 2\vec{v})$

$= 3\left((5,3,-2) + 2(2,-4,1)\right)$

$= 3\left((5,3,-2) + (4,-8,2)\right) = 3(9,-5,0) = (27,-15,0)$

(2 points) c. Find $\vec{z}$, where $2\vec{u} + \vec{z} = \vec{v}$

$\vec{z} = \vec{v} - 2\vec{u}$

$= (2,-4,1) - 2(5,3,-2)$

$= (2,-4,1) + (-10,-6,4)$

$\vec{z} = (-8,-10,5)$

(2 points) *3.* Show why $V = \left\{ \begin{bmatrix} a & b \\ 1 & c \end{bmatrix} \in M_{2,2} \middle| a,b,c \in \mathbb{R} \right\}$ with the usual operations of addition and scalar multiplication in $M_{2,2}$ is not a vector space.

$$\begin{bmatrix} a_1 & b_1 \\ 1 & c_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ 1 & c_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ 2 & c_1 + c_2 \end{bmatrix} \notin V$$

So, V is not closed under vector addition.

$\therefore$ V is not a vector space.

(4 points) *4.* Show that for any two vectors $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ in $\mathbb{R}^3$, $\vec{u} + \vec{v} = \vec{v} + \vec{u}$.

$$\begin{aligned} \vec{u} + \vec{v} &= (u_1, u_2, u_3) + (v_1, v_2, v_3) \\ &= (u_1 + v_1, u_2 + v_2, u_3 + v_3) \\ &= (v_1 + u_1, v_2 + u_2, v_3 + u_3) \\ &= (v_1, v_2, v_3) + (u_1, u_2, u_3) \\ &= \vec{v} + \vec{u} \end{aligned}$$