MATH 260 – QUIZ #4 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

1. Given the following vectors: $\vec{u} = (3,5,-2)$, $\vec{v} = (8,9,4)$

Evaluate the following.

(1 point) a. $5(-2\vec{u}+3\vec{v})$

$5[-2(3,5,-2)+3(8,9,4)]$
$= 5[(-6,-10,4)+(24,27,12)]$
$= 5(18,17,16)$
$= (90,85,80)$

(1 point) b. Find $\vec{z}$, where $4\vec{u}+\vec{z}=5\vec{v}$

$z = 5\vec{v}-4\vec{u}$
$z = 5(8,9,4)-4(3,5,-2)$
$= (40,45,20)-(12,20,-8)$
$= (28,25,28)$

(2 points) 2. Show why $V=\{A \in M_{2,2} \mid \det A = 0\}$ with the usual operations of addition and scalar multiplication in $M_{2,2}$ is not a vector space.

$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$\det A_1 = 0$ $\det A_2 = 0$

So, $A_1 \in V$ and $A_2 \in V$.

However, $A_1 + A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\det(A_1+A_2) = 1$

So, $A_1 + A_2 \notin V$.

$\therefore$ V is not a subspace of $M_{2,2}$.

(2 points) 3. Show that for any two vectors $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ in $\mathbb{R}^3$, $c(\vec{u}+\vec{v}) = c\vec{u} + c\vec{v}$.

$$c(\vec{u}+\vec{v}) = c\left((u_1,u_2,u_3) + (v_1,v_2,v_3)\right)$$
$$= c(u_1+v_1, u_2+v_2, u_3+v_3)$$
$$= (c(u_1+v_1), c(u_2+v_2), c(u_3+v_3))$$
$$= (cu_1+cv_1, cu_2+cv_2, cu_3+cv_3)$$
$$= (cu_1, cu_2, cu_3) + (cv_1, cv_2, cv_3)$$
$$= c(u_1, u_2, u_3) + c(v_1, v_2, v_3)$$
$$= c\vec{u} + c\vec{v}$$

(4 points) 4. Show that the set $S = \left\{ \begin{bmatrix} a & b \\ c & a \end{bmatrix} \middle| a,b,c \in M_2(\mathbb{R}) \right\}$ is a subspace of $M_2(\mathbb{R})$.

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & a_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & b_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & a_1+a_2 \end{bmatrix} \in S$$ $\therefore$ S is closed under addition

$$k \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & ka \end{bmatrix} \in S$$ $\therefore$ S is closed under scalar multiplication

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in S$$ $\therefore$ S is nonempty

$\therefore$ S is a subspace of $M_{2 2}$.