MATH 260 – QUIZ #4 Name: KEY

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(3 points) 1. Determine if $S=\{(5,6,5),(2,1,-5),(0,-4,1)\}$ spans $\mathbb{R}^3$.

$$c_1(5,6,5)+c_2(2,1,-5)+c_3(0,-4,1)=(v_1,v_2,v_3)$$

$$\begin{vmatrix} 5 & 2 & 0 \\ 6 & 1 & -4 \\ 5 & -5 & 1 \end{vmatrix} = 5(1-20)-2(6+20)$$
$$=5(-19)-2(26)$$
$$=-95-52$$
$$=-147\neq 0$$

$\therefore S$ spans $\mathbb{R}^3$

(3 points) 2. Determine if $S=\{-2-x,2+3x+x^2,6+5x+x^2\}$ is linearly independent or linearly dependent.

$$c_1(-2-x)+c_2(2+3x+x^2)+c_3(6+5x+x^2)=0+0x+0x^2$$

$$-2c_1+2c_2+6c_3=0$$
$$-c_1+3c_2+5c_3=0$$
$$c_2+c_3=0$$

$$\begin{vmatrix} -2 & 2 & 6 \\ -1 & 3 & 5 \\ 0 & 1 & 1 \end{vmatrix} = -1(-10+6)+1(-6+2)$$
$$=-1(-4)+1(-4)$$
$$=4-4=0$$

$\therefore S$ is linearly dependent.

(3 points) 3. Determine if $S=\{(4,-3,6,2),(1,8,3,1),(3,-2,-1,0)\}$ is linearly independent or linearly dependent.

$$c_1(4,-3,6,2)+c_2(1,8,3,1)+c_3(3,-2,-1,0)=(0,0,0,0)$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 0 \\ -3 & 8 & -2 & 0 \\ 6 & 3 & -1 & 0 \\ 2 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_1+R_2\to R_2} \begin{bmatrix} 4 & 1 & 3 & 0 \\ 1 & 9 & 1 & 0 \\ 6 & 3 & -1 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix} \xrightarrow[\substack{-6R_2+R_3\to R_3 \\ -2R_2+R_4\to R_4}]{-4R_2+R_1\to R_1} \begin{bmatrix} 0 & -35 & -1 & 0 \\ 1 & 9 & 1 & 0 \\ 0 & -51 & -7 & 0 \\ 0 & -17 & -1 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1\leftrightarrow R_2} \begin{bmatrix} 1 & 9 & 1 & 0 \\ 0 & -35 & -1 & 0 \\ 0 & -51 & -7 & 0 \\ 0 & -17 & -1 & 0 \end{bmatrix} \xrightarrow{-3R_4+R_3\to R_3} \begin{bmatrix} 1 & 9 & 1 & 0 \\ 0 & -35 & -1 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & -17 & -1 & 0 \end{bmatrix} \xrightarrow[-\frac{1}{4}R_3]{-\frac{1}{35}R_2} \begin{bmatrix} 1 & 9 & 1 & 0 \\ 0 & 1 & 1/35 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -17 & -1 & 0 \end{bmatrix} \xrightarrow{17R_2+R_4\to R_4}$$

$$\begin{bmatrix} 1 & 9 & 1 & 0 \\ 0 & 1 & 1/35 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -18/35 & 0 \end{bmatrix} \xrightarrow{\frac{18}{35}R_3+R_4\to R_4} \left[\begin{array}{ccc|c} 1 & 9 & 1 & 0 \\ 0 & 1 & 1/35 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$c_1+9c_2+c_3=0$$
$$c_2+\tfrac{1}{35}c_3=0$$
$$c_3=0$$

$\Rightarrow c_1=0,\ c_2=0,\ c_3=0$

$\therefore S$ is linearly independent

(3 points) 4. Determine if $S = \{(1,-1,0),(1,0,-1),(-1,-2,3)\}$ is a basis for $\mathbb{R}^3$.

$c_1(1,-1,0) + c_2(1,0,-1) + c_3(-1,-2,3) = (0,0,0)$ or (v_1, v_2, v_3)

$$\begin{vmatrix} 1 & 1 & -1 \\ -1 & 0 & -2 \\ 0 & -1 & 3 \end{vmatrix} = 1(0-2) - (-1)(3-1)$$
$$= -2 + 2 = 0$$

$\therefore$ S is not linear independent and S does not span $\mathbb{R}^3$

$\therefore$ S is not a basis for $\mathbb{R}^3$.

5. The following sets of vectors do not form a basis for V. Explain why not.

(2 points) b. $V = \mathbb{R}^3$; $S = \{(1,0,1),(0,1,1),(2,0,2)\}$

$(2,0,2)$ is a multiple of $(1,0,1)$.

$\therefore$ S is not linearly independent.

(2 points) c. $V = \mathbb{R}^4$; $\{(1,1,-1,2),(1,0,1,-1),(2,-1,1,-1)\}$

S has 3 vectors. In order for S to span $\mathbb{R}^4$, S must have 4 vectors.

$\therefore$ S does not span $\mathbb{R}^4$.