MATH 260 – QUIZ #5 *Name:* KEY

Directions: This is a take home quiz. This quiz is due at the beginning of class on Monday, February 11, 2019. Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

1. Given the following matrix *A*.

$$A = \begin{bmatrix} 1 & -2 & 7 & 5 \\ -2 & -1 & -9 & -7 \\ 1 & 13 & -8 & -4 \end{bmatrix}$$

(2 points) a. Determine rowspace(A).

$$\begin{bmatrix} 1 & -2 & 7 & 5 \\ -2 & -1 & -9 & -7 \\ 1 & 13 & -8 & -4 \end{bmatrix} \xrightarrow[-1R_1+R_3 \to R_3]{2R_1+R_2 \to R_2} \begin{bmatrix} 1 & -2 & 7 & 5 \\ 0 & -5 & 5 & 3 \\ 0 & 15 & -5 & -9 \end{bmatrix} \xrightarrow{3R_2+R_3 \to R_3} \begin{bmatrix} 1 & -2 & 7 & 5 \\ 0 & -5 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{-1/5 R_2} \begin{bmatrix} 1 & -2 & 7 & 5 \\ 0 & 1 & -1 & -3/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\{s(1,-2,7,5)+t(0,-5,5,3) \mid s,t, \in \mathbb{R}\}$$

or

$$\{s(1,-2,7,5)+t(0,1,-1,-3/5) \mid s,t \in \mathbb{R}\}$$

(2 points) b. Determine columnspace(A).

$$\left\{ q\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + r\begin{bmatrix} -2 \\ -1 \\ 13 \end{bmatrix} \,\middle|\, q,r \in \mathbb{R} \right\}$$

(2 points) c. Determine nullspace(A).

$$x_1 - 2x_2 + 7x_3 + 5x_4 = 0$$
$$x_2 - x_3 - \tfrac{3}{5}x_4 = 0$$

$$x_4 = p$$
$$x_3 = n$$

$$x_2 = n + \tfrac{3}{5}p$$
$$x_1 = 2n + \tfrac{6}{5}p - 7n - 5p$$
$$= -5n + \tfrac{1}{5}p$$

$$\{n(-5,1,1,0)+p(-\tfrac{19}{5}, \tfrac{3}{5}, 0, 1) \mid n,p \in \mathbb{R}\}$$

(1 point) c. Determine rank(A).

2

(1 point) e. Determine nullity(A).

2

(3 points) *2.* Find the change-of-basis matrix P from B to C where B is the ordered basis $\{(7,-4),(0,5)\}$ in $\mathbb{R}^2$ and C is the ordered basis $\{(1,-2),(2,1)\}$.

$$\left[\begin{array}{cc|cc} 1 & 2 & 7 & 0 \\ -2 & 1 & -4 & 5 \end{array}\right] \xrightarrow{2R_1+R_2 \to R_2} \left[\begin{array}{cc|cc} 1 & 2 & 7 & 0 \\ 0 & 5 & 10 & 5 \end{array}\right] \xrightarrow{\frac{1}{5}R_2} \left[\begin{array}{cc|cc} 1 & 2 & 7 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right]$$

$$\xrightarrow{-2R_2+R_1 \to R_1} \left[\begin{array}{cc|cc} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & 1 \end{array}\right]$$

$$P = \begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}$$

3. Given the mapping $T:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x_1,x_2,x_3)=(x_1+3x_2+x_3, x_1-x_2)$ is a linear transformation.

(1 point) *a.* Determine the matrix of T.

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

(2 points) *b.* Determine $\text{Kernel}(T)$.

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \xrightarrow{-1R_1+R_2 \to R_2} \begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -1 \end{bmatrix} \xrightarrow{-\frac{1}{4}R_2} \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 1/4 \end{bmatrix}$$

$x_1 + 3x_2 + x_3 = 0$

$x_2 + \frac{1}{4}x_3 = 0$

$x_3 = t$

$x_2 = -\frac{1}{4}t \qquad x_1 = +\frac{3}{4}t - t$

$x_1 = -\frac{1}{4}t$

$\{t(-\frac{1}{4}, -\frac{1}{4}, 1) \mid t \in \mathbb{R}\}$

(2 points) *c.* Determine $\text{Range}(T)$.

$$\left\{ s\begin{bmatrix} 1 \\ 1 \end{bmatrix} + r\begin{bmatrix} 3 \\ -1 \end{bmatrix} \;\middle|\; s,r \in \mathbb{R} \right\}$$