(3pts)

1. $P(2,-6,3)$ $Q(4,1,-2)$

$$d=\sqrt{(4-2)^2+(1-(-6))^2+(-2-3)^2}$$

$$=\sqrt{2^2+(7)^2+(-5)^2}$$

$$=\sqrt{4+49+25}$$

$$=\sqrt{78}$$

(3pts)

2. $P(4,-6,2)$ $Q(8,-2,3)$

$\vec{PQ}=\langle 4,4,1\rangle$

$x=4+4t$

$y=-6+4t$

$z=2+t$

3. $\vec{a}=\langle 3,-1,5\rangle, \vec{b}=\langle -4,2,1\rangle$

$\vec{c}=\langle 6,1,3\rangle$

(2pts) a. $3\vec{a}-4\vec{b}=3\langle 3,-1,5\rangle-4\langle -4,2,1\rangle$

$=\langle 9,-3,15\rangle+\langle 16,-8,-4\rangle$

$=\langle 25,-11,11\rangle$

(2pts) b. $|\vec{a}|=\sqrt{9+1+25}=\sqrt{35}$

(3pts) c. $\vec{a}\cdot\vec{c}=18-1+15=32$

(2pts) d. $\dfrac{\vec{c}}{|\vec{c}|}=\dfrac{\langle 6,1,3\rangle}{\sqrt{36+1+9}}=\left\langle \dfrac{6}{\sqrt{46}},\dfrac{1}{\sqrt{46}},\dfrac{3}{\sqrt{46}}\right\rangle$

(4pts) e. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 5\\ -4 & 2 & 1\end{vmatrix}$

$=\hat{i}(-1-10)-\hat{j}(3+20)+\hat{k}(6-4)$

$=-11\hat{i}-23\hat{j}+2\hat{k}$

1. $P(5,2,-3)$ $Q(-1,4,2)$

$$d=\sqrt{(-1-5)^2+(4-2)^2+(2-(-3))^2}$$

$$=\sqrt{(-6)^2+(2)^2+(5)^2}$$

$$=\sqrt{36+4+25}$$

$$=\sqrt{65}$$

2. $P(7,3,-2)$ $Q(3,5,-1)$

$\vec{PQ}=\langle -4,2,1\rangle$

$x=7-4t$

$y=3+2t$

$z=-2+t$

3. $\vec{a}=\langle 2,-4,1\rangle$ $\vec{b}=\langle 5,3,-6\rangle$ $\vec{c}=\langle 4,2,3\rangle$

a. $5\vec{a}-2\vec{c}=5\langle 2,-4,1\rangle-2\langle 4,2,3\rangle$

$=\langle 10,-20,5\rangle+\langle -8,-4,-6\rangle$

$=\langle 2,-24,-1\rangle$

b. $|\vec{c}|=\sqrt{16+4+9}=\sqrt{29}$

c. $\vec{b}\cdot\vec{c}=20+6-18=8$

d. $\dfrac{\vec{b}}{|\vec{b}|}=\dfrac{\langle 5,3,-6\rangle}{\sqrt{25+9+36}}=\left\langle \dfrac{5}{\sqrt{70}},\dfrac{3}{\sqrt{70}},\dfrac{-6}{\sqrt{70}}\right\rangle$

e. $\vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 1\\ 4 & 2 & 3\end{vmatrix}$

$=\hat{i}(-12-2)-\hat{j}(6-4)+\hat{k}(4+16)$

$=-14\hat{i}-2\hat{j}+20\hat{k}$

(3pts) 3 f. $\text{proj}_{\vec{b}}\vec{a} = \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\dfrac{\vec{b}}{|\vec{b}|}$

$= \left(\dfrac{-12-2+5}{16+4+1}\right)\langle -4,2,1\rangle$

$= \dfrac{-9}{21}\langle -4,2,1\rangle$

$= \left\langle \dfrac{36}{21}, -\dfrac{18}{21}, -\dfrac{9}{21} \right\rangle$

(4pts) g. $\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 2 & 1 \\ 6 & 1 & 3 \end{vmatrix}$

$= \hat{i}(6-1) - \hat{j}(-12-6) + \hat{k}(-4-12)$

$= 5\hat{i} + 18\hat{j} - 16\hat{k}$ $\quad |\vec{b}\times\vec{c}| = 25+324+256$

$= \sqrt{605}$

(4pts) 4.) $P(3,5,-1)$ $Q(1,4,2)$ $R(-2,1,5)$

$\vec{PQ} = \langle -2,-1,3\rangle$

$\vec{PR} = \langle -5,-4,6\rangle$

$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -1 & 3 \\ -5 & -4 & 6 \end{vmatrix}$

$= \hat{i}(-6+12) - \hat{j}(-12+15) + \hat{k}(8-5)$

$= 6\hat{i} - 3\hat{j} + 3\hat{k}$

$6(x-3) - 3(y-5) + 3(z+1) = 0$

$6x - 18 - 3y + 15 + 3z + 3 = 0$

$6x - 3y + 3z = 0$

$2x - y + z = 0$

f. $\text{proj}_{\vec{c}}\vec{a} = \left(\dfrac{\vec{a}\cdot\vec{c}}{|\vec{c}|}\right)\dfrac{\vec{c}}{|\vec{c}|}$

$= \left(\dfrac{8-8+3}{16+4+9}\right)\langle 4,2,3\rangle$

$= \dfrac{3}{29}\langle 4,2,3\rangle$

$= \left\langle \dfrac{12}{29}, \dfrac{6}{29}, \dfrac{9}{29} \right\rangle$

g. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 1 \\ 5 & 3 & -6 \end{vmatrix}$

$= \hat{i}(24-3) - \hat{j}(-12-5) + \hat{k}(6+20)$

$= 21\hat{i} + 17\hat{j} + 26\hat{k}$

$|\vec{a}\times\vec{b}| = 441 + 289 + 676 = \sqrt{1406}$

4. $P(-2,5,3)$ $Q(1,4,-3)$ $R(4,2,-1)$

$\vec{PQ} = \langle 3,-1,-6\rangle$

$\vec{PR} = \langle 6,-3,-4\rangle$

$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -6 \\ 6 & -3 & -4 \end{vmatrix}$

$= \hat{i}(4-18) - \hat{j}(-12+36) + \hat{k}(-9+6)$

$= -14\hat{i} - 24\hat{j} - 3\hat{k}$

$-14(x+2) - 24(y-5) - 3(z-3) = 0$

$-14x - 28 - 24y + 120 - 3z + 9 = 0$

$-14x - 24y - 3z + 101 = 0$

(4pts)
5.) 20 lb 35° 80 ft

$$\text{Work} = \vec{F}\cdot\vec{D} = |\vec{F}||\vec{D}|\cos\theta$$

$$= (20\text{ lb})(80\text{ ft})\cos 35°$$

$$\doteq 1310.6 \text{ ft-lb}$$

5.) 30 lb 60 ft 25°

$$\text{Work} = \vec{F}\cdot\vec{D} = |\vec{F}||\vec{D}|\cos\theta$$

$$= (30\text{ lb})(60\text{ ft})\cos 25°$$

$$\doteq 1631.4 \text{ ft-lb.}$$

(4pts)
6.) $P(1,4,2)$ $3x+2y+4z=6$

$P_0(2,0,0)$ is on the plane

$\overrightarrow{P_0P} = \langle -1,4,2\rangle$ $\vec{n} = \langle 3,2,4\rangle$

$$d = \left|\frac{\overrightarrow{P_0P}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{-3+8+8}{\sqrt{9+4+16}}\right| = \frac{13}{\sqrt{29}}$$

6.) $P(2,3,1)$ $3x+2y+4z=6$

$(2,0,0)$ is on the plane

$\overrightarrow{P_0P} = \langle 0,3,1\rangle$ $\vec{n} = \langle 3,2,4\rangle$

$$d = \left|\frac{\overrightarrow{P_0P}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{0+6+4}{\sqrt{9+4+16}}\right| = \frac{10}{\sqrt{29}}$$

(4pts) 7.) $\vec{a} = \langle 3,6,-1\rangle$ $\vec{b} = \langle 1,4,2\rangle$ $\vec{c} = \langle -1,3,4\rangle$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 3 & 6 & -1 \\ 1 & 4 & 2 \\ -1 & 3 & 4 \end{vmatrix}$$

$$= 3(16-6) - 6(4+2) + (-1)(3+4)$$

$$= 3(10) - 6(6) - 1(7)$$

$$= 30 - 36 - 7 = -13$$

Volume = 13

7.) $\vec{a} = \langle 1,5,-3\rangle$ $\vec{b} = \langle 3,1,-2\rangle$ $\vec{c} = \langle 2,4,3\rangle$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 1 & 5 & -3 \\ 3 & 1 & -2 \\ 2 & 4 & 3 \end{vmatrix}$$

$$= 1(3+8) - 5(9+4) + (-3)(12-2)$$

$$= 1(11) - 5(13) - 3(10)$$

$$= 11 - 65 - 30 = -84$$

Volume = 84

4pts. 8.) $\vec{r}(t) = \sqrt{t^2+48}\,\hat{\imath} + \ln(t^2+48)\,\hat{\jmath} + t\,\hat{k}$

$$\vec{r}\,'(t) = \tfrac{1}{2}(t^2+48)^{-1/2}(2t)\,\hat{\imath} + \frac{2t}{t^2+48}\,\hat{\jmath} + \hat{k}$$

$$\vec{r}\,'(1) = \frac{1}{7}\hat{\imath} + \frac{2}{49}\hat{\jmath} + \hat{k}$$

$\vec{r}(1) = 7\hat{\imath} + \ln 49\,\hat{\jmath} + \hat{k}$ $P(7, \ln 49, 1)$

$$x = 7 + \tfrac{1}{7}t,\ y = (\ln 49) + \tfrac{2}{49}t,\ z = 1+t$$

8.) $\vec{r}(t) = \sqrt{t^2+35}\,\hat{\imath} + \ln(t^2+35)\,\hat{\jmath} + t\,\hat{k}$

$$\vec{r}\,'(t) = \tfrac{1}{2}(t^2+35)^{-1/2}(2t)\,\hat{\imath} + \frac{2t}{t^2+35}\,\hat{\jmath} + \hat{k}$$

$$\vec{r}\,'(1) = \frac{1}{6}\hat{\imath} + \frac{2}{36}\hat{\jmath} + \hat{k}$$

$\vec{r}(1) = 6\hat{\imath} + \ln 36\,\hat{\jmath} + \hat{k}$ $P(6, \ln 36, 1)$

$$x = 6 + \tfrac{1}{6}t,\ y = (\ln 36) + \tfrac{1}{18}t,\ z = 1+t$$

(3pts) 9.) $\vec{r}(t) = \langle \cos 2t, t^2+1, e^{2t}+3 \rangle$

$\lim_{t \to 0} \vec{r}(t) = \langle 1, 1, 4 \rangle$

9.) $\vec{r}(t) = \langle t^3+1, \sin 2t, e^{2t}+3 \rangle$

$\lim_{t \to 0} \vec{r}(t) = \langle 1, 0, 4 \rangle$

(6pts) 10.) $\vec{r}(t) = 6\hat{\imath} + t^2\hat{\jmath} + \frac{1}{9}t^3\hat{k} \qquad 0 \le t \le 1$

$\vec{r}\,'(t) = 2t\hat{\jmath} + \frac{1}{3}t^2\hat{k}$

$|\vec{r}\,'(t)| = \sqrt{4t^2 + \frac{1}{9}t^4} = \sqrt{t^2(4+\frac{1}{9}t^2)} = t\sqrt{4+\frac{1}{9}t^2}$

$L = \int_0^1 |\vec{r}\,'(t)|\, dt = \int_0^1 t\sqrt{4+\frac{1}{9}t^2}\, dt = \frac{9}{2}\int_0^1 u^{1/2}\, du = \frac{9}{2}\frac{2}{3}u^{3/2}\Big|_0^1$

$u = 4 + \frac{1}{9}t^2$

$du = \frac{2}{9}t\, dt$

$\frac{9}{2}du = t\,dt$

$= 3\left(4+\frac{1}{9}t^2\right)^{3/2}\Big|_0^1 = 3\left(\frac{37}{9}\right)^{3/2} - 3(4)^{3/2} = \frac{1}{9}(37)^{3/2} - 3\cdot 8 = \frac{1}{9}(37)^{3/2} - 24$

11.) $\vec{r}(t) = (6t^2)\hat{\imath} + (\sin t - t\cos t)\hat{\jmath} + (\cos t + t\sin t)\hat{k}$

a.) (4pts) $\vec{r}\,'(t) = (12t)\hat{\imath} + (\cos t - \cos t + t\sin t)\hat{\jmath} + (-\sin t + \sin t + t\cos t)\hat{k}$

$= (12t)\hat{\imath} + (t\sin t)\hat{\jmath} + (t\cos t)\hat{k}$

$|\vec{r}\,'(t)| = \sqrt{144t^2 + t^2\sin^2 t + t^2\cos^2 t} = t\sqrt{145}$

$\hat{T}(t) = \frac{12}{\sqrt{145}}\hat{\imath} + \frac{\sin t}{\sqrt{145}}\hat{\jmath} + \frac{\cos t}{\sqrt{145}}\hat{k}$

b.) (4pts) $\hat{T}'(t) = \frac{\cos t}{\sqrt{145}}\hat{\jmath} + \frac{-\sin t}{\sqrt{145}}\hat{k}$

$|\hat{T}'(t)| = \sqrt{\frac{\cos^2 t}{145} + \frac{\sin^2 t}{145}} = \frac{1}{\sqrt{145}}$

$\hat{N}(t) = \frac{\hat{T}'(t)}{|\hat{T}'(t)|} = \cos t\,\hat{\jmath} - \sin t\,\hat{k}$

11 c. (4pts) $\hat{B} = \hat{T} \times \hat{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{12}{\sqrt{145}} & \frac{\sin t}{\sqrt{145}} & \frac{\cos t}{\sqrt{145}} \\ 0 & \cos t & -\sin t \end{vmatrix}$

$$= \hat{i}\left(-\frac{\sin^2 t}{\sqrt{145}} - \frac{\cos^2 t}{\sqrt{145}}\right) + \hat{j}\,\frac{12}{\sqrt{145}}\sin t + \hat{k}\,\frac{12}{\sqrt{145}}\cos t$$

$$= \left(-\frac{1}{\sqrt{145}}\right)\hat{i} + \left(\frac{12}{\sqrt{145}}\sin t\right)\hat{j} + \left(\frac{12}{\sqrt{145}}\cos t\right)\hat{k}$$

3pts

d.) $K = \frac{|\hat{T}'(t)|}{|\vec{r}'(t)|} = \frac{\frac{1}{\sqrt{145}}}{t\sqrt{145}} = \frac{1}{145t}$

(4pts)

12.) $\int\left[\frac{1}{t^4}\hat{i} + (\cos 4t)\hat{j} + (e^{5t})\hat{k}\right]dt$

$= -\frac{1}{3t^3}\hat{i} + \frac{\sin 4t}{4}\hat{j} + \frac{e^{5t}}{5}\hat{k} + \vec{C}$

$\int\left[\frac{1}{t^3}\hat{i} + (\sin 2t)\hat{j} + (e^{3t})\hat{k}\right]dt$

$= -\frac{1}{2t^2}\hat{i} - \frac{\cos 2t}{2}\hat{j} + \frac{e^{3t}}{3}\hat{k} + \vec{C}$

(4pts)

13.) $\int_0^2 \left[(t^2+3)\hat{i} + (\sin t)\hat{j} + (e^{2t})\hat{k}\right]dt$

$= \left[\left(\frac{t^3}{3} + 3t\right)\hat{i} + (-\cos t)\hat{j} + \left(\frac{e^{2t}}{2}\right)\hat{k}\right]\Big|_0^2$

$= \left[\left(\frac{8}{3}+6\right)\hat{i} + (-\cos 2)\hat{j} + \left(\frac{e^4}{2}\right)\hat{k}\right] - \left[0\hat{i} - 1\hat{j} + \frac{1}{2}\hat{k}\right]$

$= \frac{26}{3}\hat{i} + (1-\cos 2)\hat{j} + \left(\frac{e^4}{2} - \frac{1}{2}\right)\hat{k}$

4pts 14.)

$\vec{a} = (7t)\hat{\imath} + (e^{t})\hat{\jmath} + (e^{-t})\hat{k}$

$\vec{v}(t) = \left(\frac{7t^2}{2}\right)\hat{\imath} + (e^{t})\hat{\jmath} + (-e^{-t})\hat{k} + \vec{C}_1$

$\vec{v}(0) = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k} = 0\hat{\imath} + \hat{\jmath} - \hat{k} + \vec{C}_1$

$3\hat{\imath} + 4\hat{\jmath} - 1\hat{k} = \vec{C}_1$

$\vec{v}(t) = \left(\frac{7t^2}{2} + 3\right)\hat{\imath} + (e^{t} + 4)\hat{\jmath} + (-e^{-t} - 1)\hat{k}$

$\vec{r}(t) = \left(\frac{7t^3}{6} + 3t\right)\hat{\imath} + (e^{t} + 4t)\hat{\jmath} + (e^{-t} - t)\hat{k} + \vec{C}_2$

$\vec{r}(0) = 4\hat{\jmath} + 2\hat{k} = 0\hat{\imath} + 1\hat{\jmath} + 1\hat{k} + \vec{C}_2$

$3\hat{\jmath} + \hat{k} = \vec{C}_2$

$\vec{r}(t) = \left(\frac{7t^3}{6} + 3t\right)\hat{\imath} + (e^{t} + 4t + 3)\hat{\jmath} + (e^{-t} - t + 1)\hat{k}$

$\vec{a} = (5t)\hat{\imath} + (e^{t})\hat{\jmath} + (e^{-t})\hat{k}$

$\vec{v}(t) = \left(\frac{5}{2}t^2\right)\hat{\imath} + (e^{t})\hat{\jmath} + (-e^{-t})\hat{k} + \vec{C}_1$

$\vec{v}(0) = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k} = 0\hat{\imath} + 1\hat{\jmath} - \hat{k} + \vec{C}_1$

$4\hat{\imath} - 3\hat{\jmath} + 4\hat{k} = \vec{C}_1$

$\vec{v}(t) = \left(\frac{5}{2}t^2 + 4\right)\hat{\imath} + (e^{t} - 3)\hat{\jmath} + (-e^{-t} + 4)\hat{k}$

$\vec{r}(t) = \left(\frac{5}{6}t^3 + 4t\right)\hat{\imath} + (e^{t} - 3t)\hat{\jmath} + (e^{-t} + 4t)\hat{k} + \vec{C}_2$

$\vec{r}(0) = 3\hat{\jmath} + 4\hat{k} = 0\hat{\imath} + \hat{\jmath} + \hat{k} + \vec{C}_2$

$2\hat{\jmath} + 3\hat{k} = \vec{C}_2$

$\vec{r}(t) = \left(\frac{5}{6}t^3 + 4t\right)\hat{\imath} + (e^{t} - 3t + 2)\hat{\jmath} + (e^{-t} + 4t + 3)\hat{k}$

15.)

a.) $\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{9} = 1$ ellipsoid

1pt

b.) $\frac{x^2}{25} + \frac{y^2}{9} = \frac{z^2}{16}$ cone

1pt

c.) $x^2 + y^2 + z^2 = 36$ sphere

1pt -

15.

a. $\frac{x^2}{16} + \frac{y^2}{4} = \frac{z^2}{25}$ cone

b. $x^2 + y^2 + z^2 = 25$ sphere

c. $\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{16} = 1$ ellipsoid