FALL 2021 EXAM #1 KEY

(3pts) #1) $P(4,5,-2)$ $Q(2,-3,1)$

$$d = \sqrt{(2-4)^2 + (-3-5)^2 + (1-(-2))^2}$$

$$d = \sqrt{(-2)^2 + (-8)^2 + (3)^2} = \sqrt{4+64+9} = \sqrt{77}$$

(3pts) #2) $P(7,3,5)$ $Q(5,-4,2)$

$$\vec{PQ} = \langle -2, -7, -3 \rangle$$

$x = 7 - 2t$

$y = 3 - 7t$

$z = 5 - 3t$

#3.) $\vec{a} = \langle 4, -3, 6 \rangle$ $\vec{b} = \langle 7, 5, -2 \rangle$ $\vec{c} = \langle -2, 3, -4 \rangle$

(2pts) a.) $6\vec{a} - 4\vec{c} = 6\langle 4,-3,6\rangle - 4\langle -2,3,-4\rangle$

$= \langle 24, -18, 36 \rangle + \langle 8, -12, 16 \rangle$

$= \langle 32, -30, 52 \rangle$

(2pts) b.) $|\vec{a}| = \sqrt{16+9+36} = \sqrt{61}$

(3pts.) c.) $\vec{b} \cdot \vec{c} = \langle 7,5,-2 \rangle \cdot \langle -2,3,-4 \rangle$

$= -14 + 15 + 8 = 9$

(2pts) d.) $|\vec{b}| = \sqrt{49+25+4} = \sqrt{78}$

$$\frac{\vec{b}}{|\vec{b}|} = \frac{\langle 7,5,-2 \rangle}{\sqrt{78}} = \left\langle \frac{7}{\sqrt{78}}, \frac{5}{\sqrt{78}}, \frac{-2}{\sqrt{78}} \right\rangle$$

(4pts) e.) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 6 \\ 7 & 5 & -2 \end{vmatrix}$

$= \hat{i}(6-30) - \hat{j}(-8-42) + \hat{k}(20+21)$

$= -24\hat{i} + 50\hat{j} + 41\hat{k}$

(3pts) f.) $\text{proj}_{\vec{a}} \vec{c} = \dfrac{\vec{a} \cdot \vec{c}}{|\vec{a}|} \dfrac{\vec{a}}{|\vec{a}|}$

$\vec{a} \cdot \vec{c} = \langle 4,-3,6 \rangle \cdot \langle -2,3,-4 \rangle = -8 - 9 - 24$
$= -41$

$$\text{proj}_{\vec{a}} \vec{c} = \frac{-41}{61} \langle 4,-3,6 \rangle = \left\langle -\frac{164}{61}, \frac{123}{61}, -\frac{246}{61} \right\rangle$$

(3pts.) g.) $|\vec{a} \times \vec{b}| = \sqrt{24^2 + 50^2 + 41^2}$

$= \sqrt{576 + 2500 + 1681} = \sqrt{4757}$

(4pts) #4) $P(5,2,-4)$ $Q(3,6,2)$ $R(6,4,-2)$

$\vec{PQ} = \langle -2, 4, 6 \rangle$ $\vec{PR} = \langle 1, 2, 2 \rangle$

$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 4 & 6 \\ 1 & 2 & 2 \end{vmatrix} = \hat{i}(8-12) - \hat{j}(-4-6) + \hat{k}(-4-4)$

$= -4\hat{i} + 10\hat{j} - 8\hat{k}$

$-4(x-5) + 10(y-2) - 8(z+4) = 0$

$-4x + 20 + 10y - 20 - 8z - 32 = 0$

$-4x + 10y - 8z - 32 = 0$

(4pts) #5) $\text{Work} = \vec{F} \cdot \vec{D} = |\vec{F}||\vec{D}| \cos\theta$

$= (60\text{ lb})(85\text{ ft}) \cos 35^\circ$

$= 4177.7$ ft-lb.

(4pts) #6) $|\vec{\tau}| = |\vec{r} \times \vec{F}| = |\vec{r}||\vec{F}| \sin\theta$

$= (30\text{N})(\tfrac{3}{5}\text{m}) \sin 25^\circ$

$= 7.6$ N$\cdot$m

(4pts) #7) $P(5,2,-3)$ $4x - 3y + 2z = 12$

$P_0(3,0,0)$ $\vec{P_0P} = \langle 2,2,-3 \rangle$ $\vec{n} = \langle 4,-3,2 \rangle$

$$d = |\text{comp}_{\vec{n}} \vec{P_0P}| = \left| \frac{\vec{P_0P} \cdot \vec{n}}{|\vec{n}|} \right|$$

$\vec{P_0P} \cdot \vec{n} = \langle 2,2,-3 \rangle \cdot \langle 4,-3,2 \rangle = 8 - 6 - 6 = -4$

$|\vec{n}| = \sqrt{16+9+4} = \sqrt{29}$

$$d = \left| \frac{-4}{29} \right| = \frac{4}{29}$$

(4pts.) #8.) $\vec{a} = \langle 3,2,5 \rangle$ $\vec{b} = \langle 5,-3,1 \rangle$ $\vec{c} = \langle 6,4,3 \rangle$

$$V = \begin{vmatrix} 3 & 2 & 5 \\ 5 & -3 & 1 \\ 6 & 4 & 3 \end{vmatrix} = 3(-9-4) - 2(15-6) + 5(20+18)$$

$= 3(-13) - 2(9) + 5(38)$

$= -39 - 18 + 190$

$= 133$ cubic units

(3pts) #9) $\vec{r}(t) = \langle 2t^2 - 4t + 3, \sqrt{t^2+5}, 4\cos t \rangle$

$\lim_{t \to 0} \vec{r}(t) = \langle 3, \sqrt{5}, 4 \rangle$

(5pts.) #10.) $\vec{r}(t) = (t^2+3t)\hat{i} + (4e^{3t})\hat{j} + (\sin 4t)\hat{k}$

$\vec{r}\,'(t) = (2t+3)\hat{i} + (12e^{3t})\hat{j} + (4\cos 4t)\hat{k}$

$\vec{r}\,'(0) = 3\hat{i} + 12\hat{j} + 4\hat{k}$

$\vec{r}(0) = 0\hat{i} + 4\hat{j} + 0\hat{k}$ $(0,4,0)$ P

$x = 0 + 3t$
$y = 4 + 12t$
$z = 0 + 4t$

(5pts) #11) $\int \left[(t^2+4)\hat{i} + (3e^{5t})\hat{j} + (\cot t)\hat{k}\right] dt$

$= \left(\frac{t^3}{3} + 4t\right)\hat{i} + \left(\frac{3}{5}e^{5t}\right)\hat{j} + \left(\ln|\sin t|\right)\hat{k} + \vec{C}$

(5pts) #12) $\int_0^1 \left[\left(\frac{1}{1+t}\right)\hat{i} + (t^2)\hat{j} + (4e^{2t})\hat{k}\right] dt$

$= \left(\ln|1+t|\hat{i} - \frac{t^3}{3}\hat{j} + 2e^{2t}\hat{k}\right)\Big|_0^1$

$= \left[(\ln 2)\hat{i} - \frac{1}{3}\hat{j} + 2e^2\hat{k}\right] - \left[0\hat{i} - 0\hat{j} + 2\hat{k}\right]$

$= (\ln 2)\hat{i} - \frac{1}{3}\hat{j} + (2e^2 + 2)\hat{k}$

(6pts) #13) $\vec{r}(t) = \langle 2t, t^2, \frac{1}{3}t^3 \rangle$

$\vec{r}\,'(t) = \langle 2, 2t, t^2 \rangle$

$|\vec{r}\,'(t)| = 4 + 4t^2 + t^4$

$\int_0^1 \sqrt{4 + 4t^2 + t^4}\, dt = \int_0^1 \sqrt{(t^2+2)^2}\, dt$

$= \int_0^1 (t^2+2)\, dt = \left(\frac{t^3}{3} + 2t\right)\Big|_0^1 = \frac{1}{3} + 2 = \frac{7}{3}$

(5pts) #14) $\vec{r}(t) = \langle t^2 - 2t, 1 + 3t, \frac{1}{3}t^3 + \frac{1}{2}t^2 \rangle$

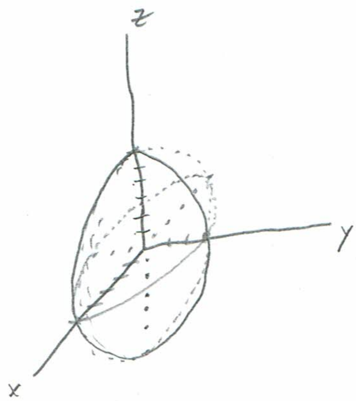
$\vec{r}\,'(t) = \langle 2t - 2, 3, t^2 + t \rangle$

$\vec{r}\,'(2) = \langle 2, 3, 6 \rangle$

$|\vec{r}\,'(2)| = \sqrt{4 + 9 + 36} = 7$

$\hat{T}(2) = \langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \rangle$

(3pts) #15) $\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{36} = 1$



(3pts.) #16.) $\frac{z}{5} = \frac{x^2}{16} + \frac{y^2}{9}$

$z = 5$ $\frac{x^2}{16} + \frac{y^2}{9} = 1$