MATH 280 – EXAM #1 **Name:** Key

Summer Session 2019

Directions: ***Please show all work for maximum credit.*** *No work = no credit.* Point values for each problem are given. There is a total of 102 points on this exam. This exam will be taken out of 100 points. Please show all work and clearly indicate your answers. Remember, this exam is to show what you know. You may not use any notes, the textbook, mobile phones, or any unauthorized sources for assistance during this exam. You may not use a calculator on this exam. Clearly indicate the answer to each question. Any work on separate paper that you would like graded must be indicated on each corresponding problem on this exam. You will need your Mt. SAC student identification card to submit this exam. Good luck!

(4 points) 1. Determine the distance between the points $P(5,-2,4)$ and $Q(-1,6,2)$.

$d = \sqrt{(-1-5)^2+(6-(-2))^2+(2-4)^2}$

$= \sqrt{(-6)^2+(8)^2+(-2)^2}$

$= \sqrt{36+64+4}$

$= \sqrt{104}$

$= \sqrt{4 \cdot 26}$

$= 2\sqrt{26}$

(4 points) 2. Show that the given equation represents a sphere, and find its center and radius.

$$x^2+y^2+z^2-2x-4y+8z-15=0$$

$x^2-2x+1+y^2-4y+4+z^2+8z+16=15+1+4+16$

$(x-1)^2+(y-2)^2+(z+4)^2=36$

center $(1,2,-4)$

radius $=6$

(5 points) *3.* Find the parametric equation form for the tangent line at the point $(1,1,0)$ to the curve traced by the vector function $\vec{r}(t) = (2-t^3)\hat{i} + (2t-1)\hat{j} + (\ln t)\hat{k}$. $t=1$

$\vec{r}'(t) = (-3t^2)\hat{i} + 2\hat{j} + \frac{1}{t}\hat{k}$

$\vec{r}'(1) = -3\hat{i} + 2\hat{j} + 1\hat{k}$

$x = 1 - 3t$

$y = 1 + 2t$

$z = t$

(5 points) *4.* Find the equation of the plane that passes through $P(2,1,5)$, $Q(4,7,-1)$, and $R(-1,4,2)$.

$\vec{PQ} = \langle 2,6,-6 \rangle$

$\vec{PR} = \langle -3,3,-3 \rangle$

$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & -6 \\ -3 & 3 & -3 \end{vmatrix} = \hat{i}(-18+18) - \hat{j}(-6-18) + \hat{k}(6+18)$

$= 24\hat{j} + 24\hat{k}$

$0(x-2) + 24(y-1) + 24(z-5) = 0$

$24y - 24 + 24z - 120 = 0$

$24y + 24z - 144 = 0$

$y + z = 6$

(5 points) *5.* A sled is pulled along a level path by a rope. A 50-lb force acting at an angle of 60° above the horizontal moves the sled 40 feet. Find the work done by the force.

$\text{Work} = \vec{F} \cdot \vec{D}$

$= |\vec{F}|\,|\vec{D}| \cos\theta$

$\text{Work} = (50\text{ lb})(40\text{ ft}) \cos 60^\circ$

$= 2000\left(\tfrac{1}{2}\right)$ ft-lb

$= 1000$ ft-lb.

6. Given the following vectors: $\bar{a} = \langle 2,4,-1\rangle$, $\bar{b} = \langle -1,3,2\rangle$, and $\bar{c} = \langle 1,2,-3\rangle$. Determine the following:

(2 points) a. $4\bar{a} - 3\bar{c}$ $= 4\langle 2,4,-1\rangle - 3\langle 1,2,-3\rangle$

$= \langle 8,16,-4\rangle + \langle -3,-6,9\rangle = \langle 5,10,5\rangle$

(2 points) b. $|\bar{b}|$ $= \sqrt{1+9+4} = \sqrt{14}$

(3 points) c. $\bar{a}\cdot\bar{b}$ $= -2+12-2 = 8$

(4 points) d. $\bar{b}\times\bar{c}$ $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 2 \\ 1 & 2 & -3 \end{vmatrix} = \hat{i}(-9-4) - \hat{j}(3-2) + \hat{k}(-2-3)$

$= -13\hat{i} - \hat{j} - 5\hat{k}$

(2 points) e. The unit vector in the direction of $\bar{b}$

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\langle -1,3,2\rangle}{\sqrt{14}} = \left\langle -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right\rangle$$

(4 points) f. $\text{proj}_{\bar{a}}\bar{b}$ $= \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}$ $\qquad |\vec{a}| = \sqrt{4+16+1} = \sqrt{21}$

$= \frac{8}{21}\langle 2,4,-1\rangle$

$= \left\langle \frac{16}{21}, \frac{32}{21}, \frac{-8}{21} \right\rangle$

(5 points) *7.* Find the area of the triangle with vertices $A(2,-3,4)$, $B(-1,-2,2)$, and $C(3,1,-3)$.

$\vec{AB} = \langle -3, 1, -2 \rangle$

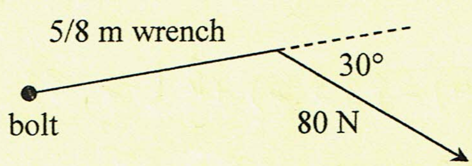
$\vec{AC} = \langle 1, 4, -7 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & -2 \\ 1 & 4 & -7 \end{vmatrix} = \hat{i}(-7+8) - \hat{j}(21+2) + \hat{k}(-12-1)$$
$$= \hat{i} - 23\hat{j} - 13\hat{k}$$

$|\vec{AB} \times \vec{AC}| = \sqrt{1+529+169}$
$= \sqrt{699}$

Area of $\Delta = \frac{1}{2}\sqrt{699}$ square units.

(5 points) *8.* A bolt is tightened by applying an 80 N force to a 5/8 meter wrench at an angle of 30° as shown below. Find the magnitude of the torque about the center of the bolt.



$|\vec{T}| = |\vec{F} \times \vec{D}|$
$= |\vec{F}||\vec{D}| \sin\theta$
$= (80\ N)(\frac{5}{8}\ m) \sin 30^\circ$
$= 50 \cdot \frac{1}{2}\ N \cdot m = 25\ J$

(4 points) *9.* Explain why if $\vec{u} \cdot \vec{v} = 0$ then $\vec{u}$ and $\vec{v}$ must be orthogonal.

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta = 0$
$\cos\theta = 0$
$\theta = \pi/2$

$\therefore$ $\vec{u}$ and $\vec{v}$ are orthogonal.

(5 points) *10.* Find the volume of the parallelepiped determined by the vectors $\vec{a} = \langle 2,-1,5\rangle$, $\vec{b} = \langle 3,1,-4\rangle$, and $\vec{c} = \langle 4,2,-1\rangle$.

$$\begin{vmatrix} 2 & -1 & 5 \\ 3 & 1 & -4 \\ 4 & 2 & -1 \end{vmatrix} = 2(-1+8)+1(-3+16)+5(6-4)$$
$$= 2(7)+1(13)+5(2)$$
$$= 14+13+10 = 37 \text{ cubic units.}$$

(4 points) 11. Evaluate the following limit: $\lim\limits_{t\to\pi/6} \vec{r}(t)$ where $\vec{r}(t) = (\csc t)\hat{i} + (\cos 2t)\hat{j} + (4t^2)\hat{k}$

$$\lim_{t\to\pi/6} \vec{r}(t) = \lim_{t\to\pi/6} (\csc t)\,\hat{i} + \lim_{t\to\pi/6} (\cos 2t)\,\hat{j} + \lim_{t\to\pi/6} (4t^2)\,\hat{k}$$
$$= 2\hat{i} + \frac{1}{2}\hat{j} + \frac{\pi^2}{9}\hat{k}$$

(5 points) 12. Evaluate the following integral: $\int_1^2 \left[(t^3)\hat{i} + \left(t\sqrt{t^2+1}\right)\hat{j} + \left(\frac{1}{t}\right)\hat{k} \right] dt$

$$\int_1^2 \left[(t^3)\hat{i} + \left(t\sqrt{t^2+1}\right)\hat{j} + \left(\frac{1}{t}\right)\hat{k}\right] dt$$
$$= \left(\int_1^2 t^3\,dt\right)\hat{i} + \left(\int_1^2 t\sqrt{t^2+1}\,dt\right)\hat{j} + \left(\int_1^2 \frac{1}{t}\,dt\right) dt$$
$$= \left(\frac{t^4}{4}\hat{i} + \frac{(t^2+1)^{3/2}}{3}\hat{j} + \ln|t|\,\hat{k}\right)\Big|_1^2$$
$$= \left(\frac{2^4}{4}\hat{i} + \frac{(2^2+1)^{3/2}}{3}\hat{j} + \ln(2)\hat{k}\right) - \left(\frac{1^4}{4}\hat{i} + \frac{(1^2+1)^{3/2}}{3}\hat{j} + \ln(1)\hat{k}\right)$$
$$= \left(4\hat{i} + \frac{5^{3/2}}{3}\hat{j} + \ln 2\,\hat{k}\right) - \left(\frac{1}{4}\hat{i} + \frac{2^{3/2}}{3}\hat{j}\right) = \frac{15}{4}\hat{i} + \left(\frac{5^{3/2}-2^{3/2}}{3}\right)\hat{j} + (\ln 2)\hat{k}$$

(5 points) *13.* Evaluate the following integral: $\int\left[(\sin 4t)\hat{\imath}+\left(3t^{7}\right)\hat{\jmath}+\left(5e^{2t}\right)\hat{k}\right]dt$

$= (-\frac{1}{4}\cos 4t)\,\hat{\imath} + (\frac{3}{8}t^{8})\,\hat{\jmath} + (\frac{5}{2}e^{2t})\,\hat{k} + \vec{C}$

(5 points) *14.* Find the length of the curve: $\vec{r}(t)=(3\cos t)\hat{\imath}+(3\sin t)\hat{\jmath}+2t^{3/2}\hat{k};\quad 0\le t\le 3$

$\vec{r}\,'(t) = (-3\sin t)\,\hat{\imath} + (3\cos t)\,\hat{\jmath} + (3t^{1/2})\,\hat{k}$

$\int_0^3 |\vec{r}\,'(t)|\,dt = \int_0^3 \sqrt{9\sin^2 t + 9\cos^2 t + 9t}\,dt = \int_0^3 \sqrt{9(1+t)}\,dt = 3\int_0^3 \sqrt{1+t}\,dt$

$= 2(1+t)^{3/2}\Big|_0^3 = 2\left[4^{3/2} - 1^{3/2}\right] = 2(8-1) = 14$ units

(5 points) *15.* Given the vectors $\vec{u}=\langle u_1,u_2,u_3\rangle$, $\vec{v}=\langle v_1,v_2,v_3\rangle$, and c a scalar. Prove the following vector property: $c(\vec{u}+\vec{v})=c\vec{u}+c\vec{v}$

$c(\vec{u}+\vec{v}) = c(\langle u_1,u_2,u_3\rangle + \langle v_1,v_2,v_3\rangle) = c\langle u_1+v_1, u_2+v_2, u_3+v_3\rangle$

$= \langle c(u_1+v_1), c(u_2+v_2), c(u_3+v_3)\rangle = \langle cu_1+cv_1, cu_2+cv_2, cu_3+cv_3\rangle$

$= \langle cu_1, cu_2, cu_3\rangle + \langle cv_1, cv_2, cv_3\rangle = c\langle u_1,u_2,u_3\rangle + c\langle v_1,v_2,v_3\rangle$

$= c\vec{u} + c\vec{v}$

(5 points) 16. Given that $\vec{r}(t) = f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k}$ where *f*, *g*, and *h* are differentiable functions of *t*. Prove that $r'(t) = f'(t)\hat{i} + g'(t)\hat{j} + h'(t)\hat{k}$.

$$\lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{(f(t+\Delta t)\hat{i} + g(t+\Delta t)\hat{j} + h(t+\Delta t)\hat{k}) - (f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k})}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \left[\frac{f(t+\Delta t) - f(t)}{\Delta t}\hat{i} + \frac{g(t+\Delta t) - g(t)}{\Delta t}\hat{j} + \frac{h(t+\Delta t) - h(t)}{\Delta t}\hat{k} \right]$$

$$= \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}\hat{i} + \lim_{\Delta t \to 0} \frac{g(t+\Delta t) - g(t)}{\Delta t}\hat{j} + \lim_{\Delta t} \frac{h(t+\Delta t) - h(t)}{\Delta t}\hat{k}$$

$$= f'(t)\hat{i} + g'(t)\hat{j} + h'(t)\hat{k}$$

17. Given the position vector $\vec{r}(t) = (6\sin 2t)\hat{i} + (6\cos 2t)\hat{j} + 5t\hat{k}$.

(5 points) a. Find the unit tangent vector, $\hat{T}(t)$.

$\vec{r}'(t) = (12\cos 2t)\hat{i} + (-12\sin 2t)\hat{j} + 5\hat{k}$ $\quad |\vec{r}'(t)| = \sqrt{144\cos^2 2t + 144\sin^2 2t + 25}$

$= \sqrt{144 + 25} = \sqrt{169} = 13$

$$\hat{T}(t) = \frac{\vec{r}'(t)}{|r'(t)|} = \frac{12}{13}\cos 2t\,\hat{i} - \frac{12}{13}\sin 2t + \frac{5}{13}\hat{k}$$

(3 points) b. Find $\hat{T}(0)$

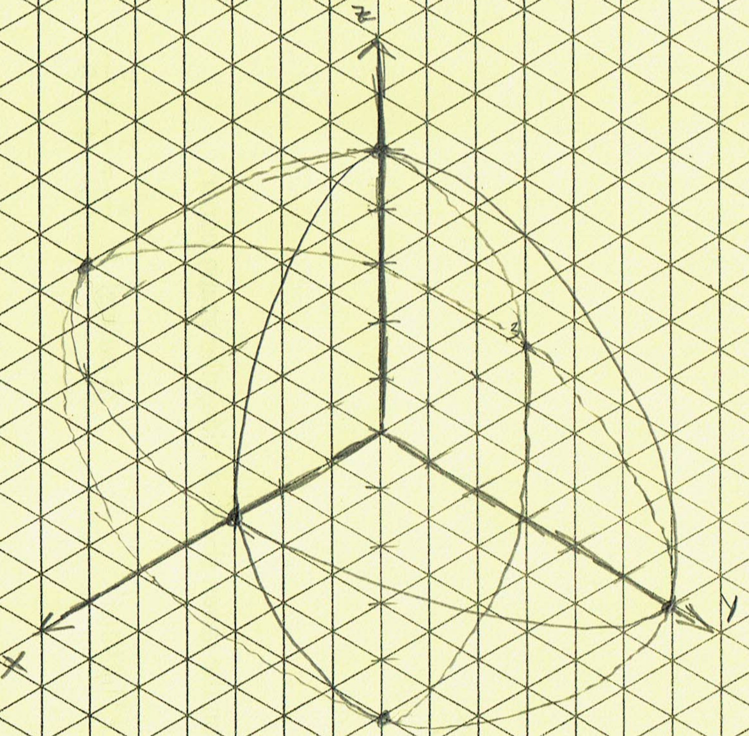
$$\hat{T}(0) = \frac{12}{13}\hat{i} + \frac{5}{13}\hat{k}$$

18. Graph the following surfaces on the given sheet of triangular graph paper.

(3 points) a. $\frac{x^2}{9} + \frac{y^2}{36} + \frac{z^2}{25} = 1$ ellipsoid

(3 points) b. $\frac{z^2}{16} = \frac{x^2}{4} + \frac{y^2}{9}$ cone

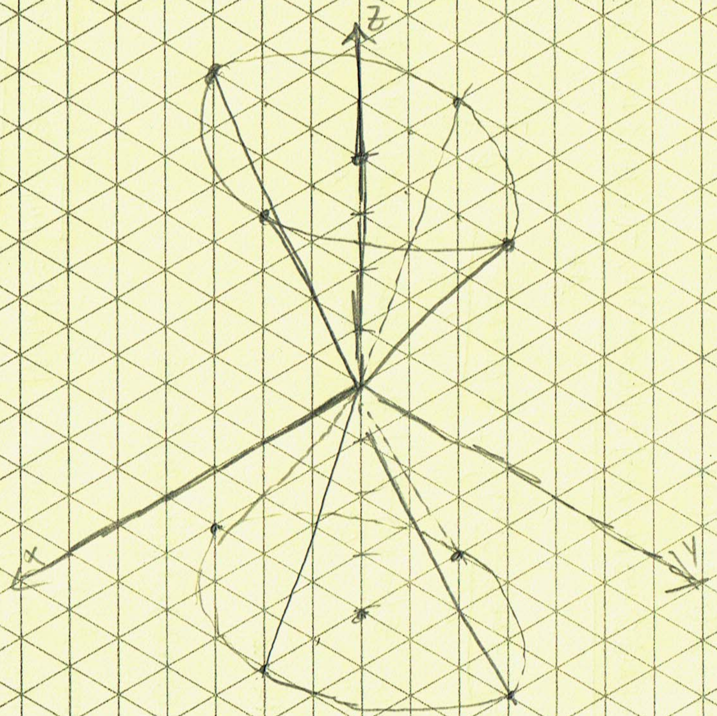
$$\frac{x^2}{9}+\frac{y^2}{36}+\frac{z^2}{25}=1$$



$$\frac{z^2}{16}=\frac{x^2}{4}+\frac{y^2}{9}$$

$z=\pm 4$

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$



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