

① $P(4, 3, -5) Q(-1, 2, 4)$

3/ $d = \sqrt{(-1-4)^2 + (2-3)^2 + (4-(-5))^2}$

$d = \sqrt{(-5)^2 + (-1)^2 + (9)^2}$

$d = \sqrt{25 + 1 + 81}$

$d = \sqrt{107}$

$P(2, -3, 6) Q(-3, 2, 4)$

$d = \sqrt{(-3-2)^2 + (2-(-3))^2 + (4-6)^2}$

$d = \sqrt{(-5)^2 + (5)^2 + (-2)^2}$

$d = \sqrt{25 + 25 + 4}$

$d = \sqrt{54} = 3\sqrt{6}$

② $P(2, 6, -3) Q(-4, 8, 2)$

$\vec{PQ} = \langle -6, 2, 5 \rangle$

$x = 2 - 6t$

$y = 6 + 2t$

$z = -3 + 5t$

$P(4, -3, 2) Q(1, 2, -4)$

$\vec{PQ} = \langle -3, 5, -6 \rangle$

$x = 4 - 3t$

$y = -3 + 5t$

$z = 2 - 6t$

③ $\vec{a} = \langle 6, 2, -5 \rangle, \vec{b} = \langle 3, -4, 1 \rangle$
 $\vec{c} = \langle 2, 4, 3 \rangle$

2/ a) $2\vec{a} - 5\vec{c} = 2\langle 6, 2, -5 \rangle - 5\langle 2, 4, 3 \rangle$
 $= \langle 12, 4, -10 \rangle + \langle -10, -20, -15 \rangle$
 $= \langle 2, -16, -25 \rangle$

2/ b) $|\vec{a}| = \sqrt{36 + 4 + 25} = \sqrt{65}$

3/ c) $\vec{a} \cdot \vec{b} = 18 - 8 - 5 = 5$

2/ d) $\frac{\vec{b}}{|\vec{b}|} = \frac{\langle 3, -4, 1 \rangle}{\sqrt{9 + 16 + 1}} = \left\langle \frac{3}{\sqrt{26}}, \frac{-4}{\sqrt{26}}, \frac{1}{\sqrt{26}} \right\rangle$

4/ e) $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 2 & 4 & 3 \end{vmatrix}$
 $= \hat{i}(-12 - 4) - \hat{j}(9 - 2) + \hat{k}(12 + 8)$
 $= -16\hat{i} - 7\hat{j} + 20\hat{k}$

$\vec{a} = \langle 4, 3, -6 \rangle, \vec{b} = \langle 6, -1, 2 \rangle$
 $\vec{c} = \langle 3, 2, 4 \rangle$

a) $4\vec{a} - 3\vec{c} = 4\langle 4, 3, -6 \rangle - 3\langle 3, 2, 4 \rangle$
 $= \langle 16, 12, -24 \rangle + \langle -9, -6, -12 \rangle$
 $= \langle 7, 6, -36 \rangle$

b) $|\vec{b}| = \sqrt{36 + 1 + 4} = \sqrt{41}$

c) $\vec{a} \cdot \vec{b} = 24 - 3 - 12 = 9$

d) $\frac{\vec{a}}{|\vec{a}|} = \frac{\langle 4, 3, -6 \rangle}{\sqrt{16 + 9 + 36}} = \left\langle \frac{4}{\sqrt{61}}, \frac{3}{\sqrt{61}}, \frac{-6}{\sqrt{61}} \right\rangle$

e) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & -6 \\ 6 & -1 & 2 \end{vmatrix}$
 $= \hat{i}(6 - 6) - \hat{j}(8 + 36) + \hat{k}(-4 - 18)$
 $= -44\hat{j} - 22\hat{k}$

3/ f.) $\text{proj}_{\vec{c}} \vec{a} = \left(\frac{\vec{a} \cdot \vec{c}}{|\vec{c}|} \right) \frac{\vec{c}}{|\vec{c}|}$

$$\vec{a} \cdot \vec{c} = 12 + 6 - 24 = -6$$

$$|\vec{c}| = \sqrt{4 + 16 + 9} = \sqrt{29}$$

$$\frac{-6}{\sqrt{29}} \langle 2, 4, 3 \rangle = \left\langle \frac{-12}{\sqrt{29}}, \frac{-24}{\sqrt{29}}, \frac{-18}{\sqrt{29}} \right\rangle$$

f.) $\text{proj}_{\vec{c}} \vec{a} = \left(\frac{\vec{a} \cdot \vec{c}}{|\vec{c}|} \right) \frac{\vec{c}}{|\vec{c}|}$

$$\vec{a} \cdot \vec{c} = 12 + 6 - 24 = -6$$

$$|\vec{c}| = \sqrt{9 + 4 + 16} = \sqrt{29}$$

$$\frac{-6}{\sqrt{29}} \langle 3, 2, 4 \rangle = \left\langle \frac{-18}{\sqrt{29}}, \frac{-12}{\sqrt{29}}, \frac{-24}{\sqrt{29}} \right\rangle$$

4/ g.)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 2 & -5 \\ 3 & -4 & 1 \end{vmatrix}$$

$$= \hat{i}(2 - 20) - \hat{j}(6 + 15) + \hat{k}(-24 - 6)$$

$$= -18\hat{i} - 21\hat{j} - 30\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{18^2 + 21^2 + 30^2}$$

$$= \sqrt{1665}$$

g.) $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -1 & 2 \\ 3 & 2 & 4 \end{vmatrix}$

$$= \hat{i}(-4 - 4) - \hat{j}(24 - 6) + \hat{k}(12 + 3)$$

$$= -8\hat{i} - 18\hat{j} + 15\hat{k}$$

$$|\vec{b} \times \vec{c}| = \sqrt{64 + 324 + 225} = \sqrt{613}$$

4/ P(3, 4, 2) Q(4, 5) R(2, -1, 4)

$$\vec{PQ} = \langle 1, -2, 7 \rangle$$

$$\vec{PR} = \langle -1, -5, 6 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 7 \\ -1 & -5 & 6 \end{vmatrix}$$

$$= \hat{i}(-12 + 35) - \hat{j}(6 + 7) + \hat{k}(-5 - 2)$$

$$= 23\hat{i} - 13\hat{j} - 7\hat{k}$$

$$23(x-3) - 13(y-4) - 7(z+2) = 0$$

$$23x - 69 - 13y + 52 - 7z - 14 = 0$$

$$23x - 13y - 7z - 31 = 0$$

P(4, 2, -1) Q(3, 5, 1) R(1, -1, 3)

$$\vec{PQ} = \langle -1, 3, 2 \rangle$$

$$\vec{PR} = \langle -3, -3, 4 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 2 \\ -3 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}(12 + 6) - \hat{j}(-4 + 6) + \hat{k}(3 + 9)$$

$$= 18\hat{i} - 2\hat{j} + 12\hat{k}$$

$$18(x-4) - 2(y-2) + 12(z+1) = 0$$

$$18x - 72 - 2y + 4 + 12z + 12 = 0$$

$$18x - 2y + 12z - 56 = 0$$

5

$$\begin{aligned} \text{Work} &= \vec{F} \cdot \vec{D} \\ &= |\vec{F}| |\vec{D}| \cos \theta \\ &= (1500 \text{ N})(3 \text{ km}) \cos 40^\circ \\ &= 3447.2 \text{ N}\cdot\text{km} \\ &= 3447199.994 \\ &= 3447200 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Work} &= \vec{F} \cdot \vec{D} \\ &= |\vec{F}| |\vec{D}| \cos \theta \\ &= (1200 \text{ N})(2 \text{ km}) \cos 50^\circ \\ &= 1542.7 \text{ N}\cdot\text{km} \\ &= 1542690.3 \text{ J} \end{aligned}$$

6

$$2x + 4y - 3z = 6$$

4

$$P_0(3, 0, 0) \quad P(3, 2, 5)$$

$$\vec{b} = \vec{P}_0 P = \langle 0, 2, 5 \rangle \quad \vec{n} = \langle 2, 4, -3 \rangle$$

$$2x + 3y - 4z = 6$$

4

$$P_0(3, 0, 0) \quad P(5, 4, 3)$$

$$\vec{b} = \vec{P}_0 P = \langle 2, 4, 3 \rangle \quad \vec{n} = \langle 2, 3, -4 \rangle$$

$$\vec{b} \cdot \vec{n} = 4 + 12 - 12 = 4$$

$$|\vec{n}| = \sqrt{4 + 9 + 16} = \sqrt{29}$$

$$\vec{b} \cdot \vec{n} = 8 - 15 = -7$$

$$|\vec{n}| = \sqrt{4 + 16 + 9} = \sqrt{29}$$

$$\text{distance} = \left| \frac{\vec{b} \cdot \vec{n}}{|\vec{n}|} \right| = \frac{7}{\sqrt{29}}$$

$$\text{distance} = \left| \frac{\vec{b} \cdot \vec{n}}{|\vec{n}|} \right| = \frac{4}{\sqrt{29}}$$

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$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 4 & -1 & 2 \\ 3 & 4 & -1 \\ 1 & 2 & 5 \end{vmatrix}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 3 & -2 & 5 \\ -1 & 3 & 2 \\ 2 & 5 & 1 \end{vmatrix}$$

4

$$\begin{aligned} &= 4(20 + 2) + 1(15 + 1) + 2(6 - 4) \\ &= 88 + 16 + 4 \\ &= 108 \end{aligned}$$

$$\begin{aligned} &= 3(3 - 10) + 2(-1 - 4) + 5(-5 - 6) \\ &= -21 - 10 - 55 \\ &= -86 \quad 86 \end{aligned}$$

8

$$\vec{r}(t) = (2 - t^3)\hat{i} + (2t - 1)\hat{j} + (2t + t)\hat{k}$$

$$\begin{aligned} \vec{r}(t) &= (2 - t^3)\hat{i} + (5t - 1)\hat{j} + (2t + t)\hat{k} \\ \vec{r}'(t) &= (-4t^3)\hat{i} + 5\hat{j} + \frac{1}{t}\hat{k} \end{aligned}$$

4

$$\vec{r}'(t) = (-3t^2)\hat{i} + (2\hat{j}) + \frac{1}{t}\hat{k}$$

$$\vec{r}'(1) = -4\hat{i} + 5\hat{j} + \hat{k}$$

$$\vec{r}(1) = -3\hat{i} + 2\hat{j} + \hat{k}$$

$$x = 1 + 3t$$

$$\vec{r}(1) = 1\hat{i} + 5\hat{j} + 0\hat{k}$$

$$x = 1 - 4t$$

$$\vec{r}(1) = 1\hat{i} + \hat{j} + 0\hat{k}$$

$$y = 1 + 2t$$

$$y = 5 + 5t$$

$$P(1, 1, 0)$$

$$z = t$$

$$P(1, 5, 0)$$

$$z = t$$

9) $\vec{v} = \langle \sqrt{2}, 1, 0 \rangle \quad \langle \sqrt{2}, 0, 0 \rangle$

10) $\vec{r}(t) = (2t^{3/2})\hat{i} + (\cos 2t)\hat{j} + (\sin 2t)\hat{k} \quad 0 \leq t \leq 1$
 $\vec{v}'(t) = (3t^{1/2})\hat{i} + (-2\sin 2t)\hat{j} + (2\cos 2t)\hat{k}$

5) $\int_0^1 \sqrt{9t + 4\sin^2 2t + 4\cos^2 2t} dt = \int_0^1 \sqrt{9t + 4} dt$ $u = 9t + 4$
 $du = 9 dt$

$$\frac{1}{9} \int_0^1 u^{1/2} du = \frac{1}{9} \cdot \frac{2}{3} u^{3/2} \Big|_0^1 = \frac{2}{27} (9t+4)^{3/2} \Big|_0^1$$

$$= \frac{2}{27} [13^{3/2} - 8]$$

11) $\vec{r}(t) = (6\sin 2t)\hat{i} + (6\cos 2t)\hat{j} + 5t\hat{k}$

a.) $\vec{v}'(t) = (12\cos 2t)\hat{i} + (-12\sin 2t)\hat{j} + 5\hat{k}$

4) $|\vec{v}'(t)| = \sqrt{144\cos^2 2t + 144\sin^2 2t + 25} = 13$

$$\hat{T}(t) = \left(\frac{12}{13} \cos 2t\right)\hat{i} + \left(-\frac{12}{13} \sin 2t\right)\hat{j} + \frac{5}{13}\hat{k}$$

b) $\hat{T}'(t) = \left(-\frac{24}{13} \sin 2t\right)\hat{i} + \left(-\frac{24}{13} \cos 2t\right)\hat{j}$

4) $|\hat{T}'(t)| = \sqrt{\left(\frac{24}{13}\right)^2 \sin^2 2t + \left(\frac{24}{13}\right)^2 \cos^2 2t} = \frac{24}{13}$

$$\hat{N}(t) = (-\sin 2t)\hat{i} + (-\cos 2t)\hat{j}$$

c) $\vec{B} = \hat{T} \times \hat{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{12}{13} \cos 2t & -\frac{12}{13} \sin 2t & \frac{5}{13} \\ -\sin 2t & -\cos 2t & 0 \end{vmatrix} = \hat{i} \left(\frac{5}{13} \cos 2t\right) - \hat{j} \left(\frac{5}{13} \sin 2t\right) + \hat{k} \left(\frac{12}{13} \cos^2 2t - \frac{12}{13} \sin^2 2t\right)$
 $= \left(\frac{5}{13} \cos 2t\right)\hat{i} + \left(-\frac{5}{13} \sin 2t\right)\hat{j} + \left(-\frac{12}{13}\right)\hat{k}$

4) d.) $\kappa = \frac{|\hat{T}'(t)|}{|\vec{v}'(t)|} = \frac{24/13}{13} = \frac{24}{169}$

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$$\int [\cot t \hat{i} + \frac{1}{t^2} \hat{j} + (\cos 3t) \hat{k}] dt$$

$$= \ln|\sin t| \hat{i} - \frac{1}{t} \hat{j} + (\frac{1}{3} \sin 3t) \hat{k} + \vec{C}$$

$$\int [\cot t \hat{i} + (\sin 4t) \hat{j} + \frac{1}{t^3} \hat{k}] dt$$

$$= \ln|\sin t| \hat{i} + (-\frac{1}{4} \cos 4t) \hat{j} + (-\frac{1}{2t^2}) \hat{k} + \vec{C}$$

13

$$\int_0^1 [(t^2 \ln(t^3+1)) \hat{i} + (e^{4t}) \hat{j} + (3t^2+1) \hat{k}] dt$$

3/

$$u = t^3$$

$$du = 3t^2 dt$$

$$\frac{1}{3} \ln u$$

$$= \left[\frac{1}{3} [u \ln u - u] \hat{i} + (\frac{1}{4} e^{4t}) \hat{j} + (t^3 + t) \hat{k} \right] \Big|_0^1$$

$$= \left[\frac{1}{3} [(1^3+0) \ln(1^3+1) - (1^3+1)] \hat{i} + (\frac{1}{4} e^4) \hat{j} + (1^3+1) \hat{k} \right] \Big|_0^1$$

$$= \frac{1}{3} [2 \ln 2 - 2 + 1] \hat{i} + [\frac{1}{4} (e^4 - 1)] \hat{j} + [2 - 0] \hat{k}$$

$$= \frac{1}{3} (2 \ln 2 - 1) \hat{i} + \frac{1}{4} (e^4 - 1) \hat{j} + 2 \hat{k}$$

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$$\vec{a} = (t^2+4) \hat{i} + t \hat{j} + 2 \hat{k}$$

$$\vec{v} = (\frac{1}{3} t^3 + 4t) \hat{i} + (\frac{1}{2} t^2) \hat{j} + (2t) \hat{k} + \vec{C}_1$$

$$\vec{v}(0) = 2 \hat{j} + 4 \hat{k} = \vec{C}_1$$

$$\vec{v} = (\frac{1}{3} t^3 + 4t) \hat{i} + (\frac{1}{2} t^2 + 2) \hat{j} + (2t + 4) \hat{k}$$

$$\vec{r} = (\frac{1}{12} t^4 + 2t^2) \hat{i} + (\frac{1}{6} t^3 + 2t) \hat{j} + (t^2 + 4t) \hat{k} + \vec{C}_2$$

$$\vec{r}(0) = \hat{i} + 3 \hat{j} = \vec{C}_2$$

$$\vec{r}(t) = (\frac{1}{12} t^4 + 2t^2 + 1) \hat{i} + (\frac{1}{6} t^3 + 2t + 3) \hat{j} + (t^2 + 4t) \hat{k}$$

$$\vec{a} = (t^2+5) \hat{i} + t \hat{j} + 3 \hat{k}$$

$$\vec{v} = (\frac{1}{3} t^3 + 5t) \hat{i} + (\frac{1}{2} t^2) \hat{j} + (3t) \hat{k} + \vec{C}_1$$

$$\vec{v}(0) = 4 \hat{j} + 2 \hat{k} = \vec{C}_1$$

$$\vec{v} = (\frac{1}{3} t^3 + 5t) \hat{i} + (\frac{1}{2} t^2 + 4) \hat{j} + (3t + 2) \hat{k}$$

$$\vec{r} = (\frac{1}{12} t^4 + \frac{5}{2} t^2) \hat{i} + (\frac{1}{6} t^3 + 4t) \hat{j} + (\frac{3}{2} t^2 + 2t) \hat{k}$$

$$\vec{r}(0) = \hat{i} + 4 \hat{j} = \vec{C}_2$$

$$\vec{r}(t) = (\frac{1}{12} t^4 + \frac{5}{2} t^2 + 1) \hat{i} + (\frac{1}{6} t^3 + 4t + 4) \hat{j} + (\frac{3}{2} t^2 + 2t) \hat{k}$$

4/

15

$$\vec{v}(t) = (80\sqrt{3})t\hat{i} + (96 + 80t - 16t^2)\hat{j}$$

3/ a.) $96 + 80t - 16t^2$

$$80 - 32t = 0$$

$$80 = 32t$$

$$t = \frac{80}{32} = 2.5 \text{ sec}$$

2/ b.) $96 + 80(2.5) - 16(2.5)^2 = 196 \text{ ft}$

c.) $96 + 80t - 16t^2 = 0$

3/ $-16(t^2 - 5t - 6) = 0$

$$-16(t - 6)(t + 1) = 0$$

$$t = 6 \text{ sec}$$

2/ d.) $(80\sqrt{3})(6) = 480\sqrt{3} \text{ ft}$

- 16) 1 j
- 2 ~~d~~
- 3 ~~d~~
- 4 a
- 5 h
- 6 b.
- 7 g
- 8 e
- 9 i
- 10 f

- 1 b
- 2 j
- 3 f
- 4 ~~d~~
- 5 i
- 6 h
- 7 ~~d~~
- 8 g
- 9 a
- 10 e