MATH 280 – EXAM #1 **Name:** KEY
Summer Session 2022

Directions: ***<u>Please show all work for maximum credit.</u>*** *No work = no credit.* Point values for each problem are given. There is a total of 90 points on this exam. Please show all work and clearly indicate your answers. Remember, this exam is to show what you know. You may use one sheet of notes for this exam. You may not use the textbook, mobile phones, or any unauthorized sources for assistance during this exam. You may not use a calculator on this exam. Clearly indicate the answer to each question. Any work on separate paper that you would like graded must be indicated on each corresponding problem on this exam. Please write your name on all pages of this exam and your scratch paper. Good luck!

(3 points) 1. Determine the distance between the points $P(3,1,-5)$ and $Q(4,-2,3)$.

$$d = \sqrt{(4-3)^2 + (-2-1)^2 + (3-(-5))^2}$$

$$= \sqrt{1+9+64} = \sqrt{74}$$

(3 points) 2. Find the parametric form of the equation of the line passing through $P(3,5,-1)$ and $Q(6,-2,4)$.

$$\vec{PQ} = \langle 3,-7,5\rangle$$

$$x = 3+3t$$
$$y = 5-7t$$
$$z = -1+5t$$

(4 points) 3. Find the equation of the plane that passes through $P(2,-1,4)$, $Q(4,5,2)$, and $R(-1,4,3)$.

$$\vec{PQ} = \langle 2,6,-2\rangle \qquad \vec{PR} = \langle -3,5,-1\rangle$$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & -2 \\ -3 & 5 & -1 \end{vmatrix} = \hat{i}(-6+10) - \hat{j}(-2-6) + \hat{k}(10+18)$$

$$= 4\hat{i} + 8\hat{j} + 28\hat{k}$$

$$4(x-2) + 8(y+1) + 28(z-4) = 0$$

$$4x - 8 + 8y + 8 + 28z - 112 = 0$$

$$4x + 8y + 28z - 112 = 0$$

4. Given the following vectors: $\vec{a} = \langle 2,5,-1 \rangle$, $\vec{b} = \langle 3,6,4 \rangle$, and $\vec{c} = \langle -3,2,5 \rangle$. Determine the following:

(2 points) a. $4\vec{b} - 3\vec{c}$

$4\langle 3,6,4\rangle - 3\langle -3,2,5\rangle$

$\langle 12,24,16\rangle + \langle +9,-6,-15\rangle$

$= \langle 21, 18, 1\rangle$

(2 points) b. $|\vec{a}|$

$|\vec{a}| = \sqrt{4+25+1}$

$= \sqrt{30}$

(3 points) c. $\vec{a} \cdot \vec{c}$

$-6+10-5 = -1$

(2 points) d. The unit vector in the direction of $\vec{c}$.

$|\vec{c}| = \sqrt{9+4+25} = \sqrt{38}$

$\hat{c} = \frac{\langle -3,2,5\rangle}{\sqrt{38}} = \langle -\frac{3}{\sqrt{38}}, \frac{2}{\sqrt{38}}, \frac{5}{\sqrt{38}}\rangle$

(4 points) e. $\vec{a} \times \vec{b}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -1 \\ 3 & 6 & 4 \end{vmatrix}$$

$= \hat{i}(20+6) - \hat{j}(8+3) + \hat{k}(12-15)$

$= 26\hat{i} - 11\hat{j} - 3\hat{k}$

(3 points) f. $\text{proj}_{\vec{b}}\vec{a}$

$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\left(\frac{\vec{b}}{|\vec{b}|}\right)$

$|\vec{b}| = \sqrt{9+36+16}$ (25)

$= \sqrt{61}$

$\vec{a}\cdot\vec{b} = 6+30-4 = 32$

$\frac{32}{61}\langle 3,6,4\rangle = \langle \frac{96}{61}, \frac{192}{61}, \frac{128}{61}\rangle$

(3 points) g. Find the area of the parallelogram defined by $\vec{a}$ and $\vec{b}$.

$|\vec{a}\times\vec{b}| = \sqrt{26^2+11^2+3^2}$

$= \sqrt{676+121+9}$

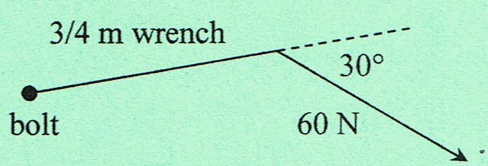
$= \sqrt{806}$

(4 points) 5. A sled is pulled along a level path by a rope. A 60-lb force acting at an angle of $40°$ above the horizontal moves the sled 90 feet. Find the work done by the force. Round your answer to the nearest tenth.

$$\text{Work} = |\vec{F}|\,|\vec{D}| \cos\theta$$

$$= (60\text{ lb})(90\text{ ft}) \cos 40° = 4136.7 \text{ ft-lb}$$

(4 points) 6. A bolt is tightened by applying an 60 N force to a 3/4 meter wrench at an angle of $30°$ as shown below. Find the magnitude of the torque about the center of the bolt.



$$|\vec{T}| = |\vec{F}|\,|\vec{D}| \sin\theta$$

$$= (60\text{ N})\left(\tfrac{3}{4}\text{ m}\right) \sin 30°$$

$$= \frac{45}{2} = 22.5 \text{ N-m}$$

(4 points) 7. Find the distance from the point $P(3,6,1)$ to the plane $2x+3y-4z=12$.

$P_0(6,0,0)$ $\quad \overrightarrow{P_0P} = \langle -3,6,1 \rangle \quad \vec{n} = \langle 2,3,-4 \rangle$

$$\text{distance} = \frac{\overrightarrow{P_0P}\cdot\vec{n}}{|\vec{n}|} = \frac{-6+18-4}{\sqrt{4+9+16}} = \frac{8}{\sqrt{29}}$$

(4 points) 8. Find the volume of the parallelepiped determined by the vectors $\vec{a}=\langle 3,1,2\rangle$, $\vec{b}=\langle -2,-1,1\rangle$, and $\vec{c}=\langle 4,2,-2\rangle$.

$$\begin{vmatrix} 3 & 1 & 2 \\ -2 & -1 & 1 \\ 4 & 2 & -2 \end{vmatrix} = 3(2-2)+1(+4-4)+2(-4+4)$$

$$=0$$

9. Evaluate the following integrals.

(4 points) a. $\int\left[(\cos 4t)\hat{i}+\left(\frac{1}{t}\right)\hat{j}+\left(5e^{3t}\right)\hat{k}\right]dt$

$$=\frac{1}{4}\sin 4t\,\hat{\imath}+\ln|t|\,\hat{\jmath}+\frac{5}{3}e^{3t}\,\hat{k}+\vec{C}$$

(4 points) b. $\int_0^1\left[\left(e^{2t}\right)\hat{i}+\left(t^3\right)\hat{j}+(\sin 5t)\hat{k}\right]dt$

$$\left(\frac{1}{2}e^{2t}\hat{\imath}+\frac{t^4}{4}\hat{\jmath}-\frac{1}{5}\cos 5t\right)\Big|_0^1$$

$$=\left(\frac{1}{2}e^2\hat{\imath}+\frac{1}{4}\hat{\jmath}-\frac{1}{5}\cos 5\hat{k}\right)-\left(\frac{1}{2}\hat{\imath}+0\hat{\jmath}-\frac{1}{5}\hat{k}\right)$$

$$=\left(\frac{1}{2}e^2-\frac{1}{2}\right)\hat{\imath}+\frac{1}{4}\hat{\jmath}+\left(-\frac{1}{5}\cos 5-\frac{1}{5}\right)\hat{k}$$

10. Given the position vector: $\vec{r}(t) = (4\cos t)\hat{i} + (3t)\hat{j} + (4\sin t)\hat{k}$.

(4 points) a. Find the unit tangent vector $\hat{T}(t)$.

$\vec{r}\,'(t) = (-4\sin t)\hat{\imath} + 3\hat{\jmath} + (4\cos t)\hat{k}$

$|\vec{r}\,'(t)| = \sqrt{16\sin^2 t + 9 + 16\cos^2 t} = 5$

$\hat{T} = (-\frac{4}{5}\sin t)\hat{\imath} + \frac{3}{5}\hat{\jmath} + (\frac{4}{5}\cos t)\hat{k}$

(4 points) a. Find the unit normal vector $\hat{N}(t)$.

$\hat{T}' = (-\frac{4}{5}\cos t)\hat{\imath} + 0\hat{\jmath} + (-\frac{4}{5}\sin t)\hat{k}$

$|\hat{T}'| = \frac{4}{5}$

$\hat{N} = (-\cos t)\hat{\imath} + 0\hat{\jmath} + (-\sin t)\hat{k}$

(4 points) a. Find the binormal vector $\hat{B}(t)$.

$$\hat{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -\frac{4}{5}\sin t & \frac{3}{5} & \frac{4}{5}\cos t \\ -\cos t & 0 & -\sin t \end{vmatrix} = \hat{\imath}(-\frac{3}{5}\sin t) - (\frac{4}{5}\sin^2 t + \frac{4}{5}\cos^2 t)\hat{\jmath} + \hat{k}(\frac{3}{5}\cos t)$$

$(-\frac{3}{5}\sin t)\hat{\imath} - \frac{4}{5}\hat{\jmath} + (\frac{3}{5}\cos t)\hat{k}$

(3 points) d. Find the curvature κ.

$\kappa = \frac{|\hat{T}'|}{|\vec{r}\,'|} = \frac{4/5}{5} = \frac{4}{25}$

11. Given that the following position vector describes a projectile in motion where distance is in feet and time is in seconds: $\vec{r}(t) = \left(32\sqrt{3}\right)t\hat{i} + \left(128 + 32t - 16t^2\right)\hat{j}$

(3 points) a. What is the maximum height that the projectile attains?

$r_y' = 32 - 32t = 0$

$t = 1 \text{ sec}$

$r_y(1) = 128 + 32 - 16 = 144 \text{ ft}$

(3 points) b. When does the projectile hit the ground?

$128 + 32t - 16t^2 = 0$

$-16(t^2 - 2t - 8) = 0$ $\quad t = 4 \text{ sec.}$

$(t-4)(t+2) = 0$

(3 points) c. What is the range of the projectile?

$(32\sqrt{3})(4) = 128\sqrt{3} \text{ ft.}$

(3 points) 12. Evaluate the following limit: $\lim_{t\to 0} \vec{r}(t)$ where $\vec{r}(t) = (\cos 3t)\hat{i} + \left(4e^{5t}\right)\hat{j} + \left(t^2 + 4t - 3\right)\hat{k}$.

$\lim_{t\to 0} \vec{r}(t) = \hat{i} + 4\hat{j} - 3\hat{k}$

(5 points) 13. Find the length of the curve: $\vec{r}(t) = (2\cos t)\hat{i} + (2\sin t)\hat{j} + t^2\hat{k}$; $0 \le t \le \pi/4$

t

$\vec{r}'(t) = (-2\sin t)\hat{i} + (2\cos t)\hat{j} + 1\hat{k}$

$\int_0^{\pi/4} \sqrt{4\sin^2 t + 4\cos^2 t + 1} \, dt$

$\int_0^{\pi/4} \sqrt{4 + 1^2} = \int_0^{\pi/4} \sqrt{5}\, dt = \sqrt{5}\, t \Big|_0^{\pi/4}$

$= \dfrac{\pi\sqrt{5}}{4}$

14. Identify the following equations. You do not need to graph them.

(1 point) a. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + \dfrac{z^2}{25} = 1$ ellipsoid

(1 point) b. $\dfrac{z}{6} = \dfrac{x^2}{25} + \dfrac{y^2}{16}$ elliptic paraboloid

(1 point) c. $\dfrac{x^2}{16} - \dfrac{y^2}{25} + \dfrac{z^2}{9} = 1$ hyperboloid of one sheet

(1 point) d. $\dfrac{z^2}{4} = \dfrac{x^2}{9} + \dfrac{y^2}{16}$ cone

(1 point) e. $x^2 + y^2 + z^2 = 25$ sphere