MATH 280 EXAM #1 KEY - WINTER 2022

(3pts)
#1) P(6,3,-5) Q(4,1,-2)

$d = \sqrt{(4-6)^2 + (1-3)^2 + (-2-(-5))^2}$

$= \sqrt{(-2)^2 + (-2)^2 + 3^2} = \sqrt{4+4+9} = \sqrt{17}$

(3pts)
#2.) P(8,-6,5) Q(2,1,9)

$\vec{PQ} = \langle -6, 7, 4 \rangle$

$x = 8 - 6t$
$y = -6 + 7t$
$z = 5 + 4t$

(4pts)
#3) P(0,-4,4) Q(1,-2,2) R(3,2,-2)

$d_{PQ} = \sqrt{(1-0)^2 + (-2-(-4))^2 + (2-4)^2} = \sqrt{1^2 + (2)^2 + (-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

$d_{QR} = \sqrt{(3-1)^2 + (2-(-2))^2 + (-2-2)^2} = \sqrt{2^2 + 4^2 + (-4)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

$d_{PR} = \sqrt{(3-0)^2 + (2-(-4))^2 + (-2-4)^2} = \sqrt{3^2 + 6^2 + (-6)^2} = \sqrt{9+36+36} = \sqrt{81} = 9$

$3+6=9$ They lie on a straight line.

(4pts)
#4.) P(4,4,5) Q(2,2,4) R(2,8,1)

$d_{PQ} = \sqrt{(2-4)^2 + (2-4)^2 + (4-5)^2} = \sqrt{(-2)^2 + (-2)^2 + (-1)^2} = \sqrt{4+4+1} = \sqrt{9}$

$d_{QR} = \sqrt{(2-2)^2 + (8-2)^2 + (1-4)^2} = \sqrt{0^2 + (-6)^2 + (-3)^2} = \sqrt{0+36+9} = \sqrt{45}$

$d_{PR} = \sqrt{(2-4)^2 + (8-4)^2 + (1-5)^2} = \sqrt{(-2)^2 + (4)^2 + (-4)^2} = \sqrt{4+16+16} = \sqrt{36}$

$\sqrt{9}^2 + \sqrt{36}^2 = \sqrt{45}^2$ It is a right triangle

$9 + 36 = 45$ ✓

It is not an isosceles triangle

#5.) $\vec{a} = \langle 7,2,5 \rangle$ $\vec{b} = \langle 3,6,-1 \rangle$ $\vec{c} = \langle 4,1,3 \rangle$

(2pts)
a.) $5\vec{a} - 3\vec{b} = 5\langle 7,2,5 \rangle - 3\langle 3,6,-1 \rangle$

$= \langle 35,10,25 \rangle + \langle -9,-18,3 \rangle = \langle 26,-8,28 \rangle$

(2pts)
b.) $|\vec{b}| = \sqrt{9+36+1} = \sqrt{46}$

(3pts)
c.) $\vec{a} \cdot \vec{c} = \langle 7,2,5 \rangle \cdot \langle 4,1,3 \rangle = 28 + 2 + 15 = 45$

(2pts)
d.) $|\vec{c}| = \sqrt{4^2+1^2+3^2} = \sqrt{16+1+9} = \sqrt{26}$

$$\frac{\vec{c}}{|\vec{c}|} = \frac{\langle 4,1,3 \rangle}{\sqrt{26}} = \left\langle \frac{4}{\sqrt{26}}, \frac{1}{\sqrt{26}}, \frac{3}{\sqrt{26}} \right\rangle$$

(4pts)
e.) $$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & -1 \\ 4 & 1 & 3 \end{vmatrix} = \hat{i}(18-(-1)) - \hat{j}(9-(-4)) + \hat{k}(3-24)$$

$$= 19\hat{i} - 13\hat{j} - 21\hat{k}$$

(3pts)
f.) $$\text{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|} = \left(\frac{28}{\sqrt{78}}\right)\frac{\langle 7,2,5 \rangle}{\sqrt{78}} = \frac{28}{78}\langle 7,2,5 \rangle = \frac{14}{39}\langle 7,2,5 \rangle$$

$$= \left\langle \frac{98}{39}, \frac{28}{39}, \frac{70}{39} \right\rangle$$

$\vec{a}\cdot\vec{b} = \langle 7,2,5 \rangle \cdot \langle 3,6,-1 \rangle = 21 + 12 - 5 = 28$

$|\vec{a}| = \sqrt{49+4+25} = \sqrt{78}$

(3pts)
g.) $|\vec{b} \times \vec{c}| = \sqrt{19^2 + (-13)^2 + (-21)^2} = \sqrt{361 + 169 + 441} = \sqrt{971}$ square units

(4pts)
#6) $P(3,5,1)$ $Q(6,2,-3)$ $R(8,4,-2)$

$\vec{PQ} = \langle 3,-3,-4 \rangle$ $\vec{PR} = \langle 5,-1,-3 \rangle$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -3 & -4 \\ 5 & -1 & -3 \end{vmatrix} = \hat{i}(9-4) - \hat{j}(-9+20) + \hat{k}(-3+15)$$

$$= 5\hat{i} - 11\hat{j} + 12\hat{k}$$

$5(x-3) - 11(y-5) + 12(z-1) = 0$

$5x - 15 - 11y + 55 + 12z - 12 = 0$

$5x - 11y + 12z + 28 = 0$

(4pts)
#7) Work $= \vec{F}\cdot\vec{D} = |\vec{F}||\vec{D}|\cos\theta$

$= (75\,lb)(120\,ft)\cos 25^0$

$= 8156.8$ ft-lb.

(4pts)
#8.) $|\vec{T}| = |\vec{r}\times\vec{F}| = |\vec{r}||\vec{F}|\sin\theta$

$= (\frac{3}{5}m)(30N)\sin 25^0$

$= 7.6$ N·m

(4pts)
#9.) $P_0\,(6,0,0)$ $\quad \overrightarrow{P_0P}\,\langle -4,4,5\rangle$ $\quad \vec{n} = \langle 2,-4,3\rangle$

$P(2,4,5)$

$$d = |\text{comp}_{\vec{n}}\,\overrightarrow{P_0P}| = \left|\frac{\overrightarrow{P_0P}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{-8-16+15}{\sqrt{4+16+9}}\right| = \left|\frac{-9}{\sqrt{29}}\right| = \frac{9}{\sqrt{29}}$$

(4pts)
#10)
$$V = \begin{vmatrix} 4 & 1 & 5 \\ 2 & -3 & 6 \\ 3 & 4 & -2 \end{vmatrix} = 4(6-24) - 1(-4-18) + 5(8+9)$$

$= 4(-18) - 1(-22) + 5(17)$

$= -72 + 22 + 85 = 35$ cubic units

(3pts)
#11)
$$\lim_{t\to 0}\vec{r}(t) = \lim_{t\to 0}\langle\sqrt{t^2+5t+9},\ 6\cos t,\ e^{4t}\rangle = \langle 3, 6, 1\rangle$$

(5pts)
#12) $\vec{r}(t) = (t^3+5t)\hat{i} + (\tan 3t)\hat{j} + (2e^{4t})\hat{k}$ $\qquad t=0 \quad \vec{r}(0) = 0\hat{i} + 0\hat{j} + 2\hat{k}$

$\vec{r}\,'(t) = (3t^2+5)\hat{i} + (3\sec^2 3t)\hat{j} + (8e^{4t})\hat{k}$ $\qquad \vec{r}\,'(0) = 5\hat{i} + 3\hat{j} + 8\hat{k}$

Equation of tangent line

$x = 0 + 5t$

$y = 0 + 3t$

$z = 2 + 8t$

(4pts)

#13)
$$\int \left[(\sin 4t)\hat{\imath} + (t^2+3t+2)\hat{\jmath} + (e^{5t})\hat{k}\right]dt$$

$$= \left(-\tfrac{1}{4}\cos 4t\right)\hat{\imath} + \left(\tfrac{1}{3}t^3 + \tfrac{3}{2}t^2 + 2t\right)\hat{\jmath} + \left(\tfrac{1}{5}e^{5t}\right)\hat{k} + \vec{C}$$

(4pts)

#14)
$$\int_0^1 \left[\left(\frac{1}{1+t}\right)\hat{\imath}\,(t^2)\hat{\jmath} + (4e^{2t})\hat{k}\right]dt$$

$$= \left[\ln(1+t)\,\hat{\imath} + \left(\tfrac{1}{3}t^3\right)\hat{\jmath} + (2e^{2t})\hat{k}\right]\Big|_0^1$$

$$= \left(\ln 2\,\hat{\imath} + \tfrac{1}{3}\hat{\jmath} + 2e^2\hat{k}\right) - \left(0\hat{\imath} + 0\hat{\jmath} + 2\hat{k}\right)$$

$$= (\ln 2)\hat{\imath} + \tfrac{1}{3}\hat{\jmath} + (2e^2 - 2)\hat{k}$$

(6pts)

#15.)
$$\vec{r}(t) = \langle 2t, t^2, \tfrac{1}{3}t^3 \rangle$$

$$\vec{r}\,'(t) = \langle 2, 2t, t^2 \rangle$$

$$L = \int_0^1 \sqrt{4+4t^2+t^4}\,dt = \int_0^1 \sqrt{(2+t^2)^2}\,dt = \int_0^1 (2+t^2)\,dt = \left(2t + \tfrac{1}{3}t^3\right)\Big|_0^1$$

$$= 2 + \tfrac{1}{3} = \tfrac{7}{3}$$

(5pts)

#16)
$$\vec{r}(t) = \langle e^{2t}, e^{-2t}, te^{2t} \rangle$$

$$\vec{r}\,'(t) = \langle 2e^{2t}, -2e^{-2t}, e^{2t} + 2te^{2t} \rangle$$

$$\vec{r}\,'(0) = \langle 2, -2, 1 \rangle$$

$$|\vec{r}\,'(0)| = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$\hat{T}(0) = \frac{\langle 2, -2, 1 \rangle}{3} = \langle \tfrac{2}{3}, -\tfrac{2}{3}, \tfrac{1}{3} \rangle$$

#17.)

a.) ellipsoid

b.) elliptic paraboloid

c.) cone

d.) sphere

e.) hyperboloid of one sheet.