MATH 280 EXAM #2 FALL 2021

#1) $\vec{r}(t) = \langle t^2, \frac{2}{3}t^3, t \rangle$ $\quad P_0(1, \frac{2}{3}, 1)$ $\quad t=1$

$\vec{r}\,'(t) = \langle 2t, 2t^2, 1 \rangle$

$|\vec{r}\,'(t)| = \sqrt{4t^2 + 4t^4 + 1} = \sqrt{(2t^2+1)^2} = 2t^2+1$

$\hat{T}(t) = \dfrac{\langle 2t, 2t^2, 1 \rangle}{2t^2+1}$

$\hat{T}'(t) = -(2t^2+1)^{-2}(4t)\langle 2t, 2t^2, 1\rangle + (2t^2+1)^{-1}\langle 2, 4t, 0\rangle$

$= -\dfrac{\langle 8t^2, 8t^3, 4t\rangle}{(2t^2+1)^2} + \dfrac{\langle 2, 4t, 0\rangle}{(2t^2+1)} = -\dfrac{\langle 8t^2, 8t^3, 4t\rangle}{(2t^2+1)^2} + \dfrac{\langle 4t^2+2, 8t^3+4t, 0\rangle}{(2t^2+1)^2}$

$= \dfrac{\langle -4t^2+2, 4t, -4t\rangle}{(2t^2+1)^2}$

$|\hat{T}'(t)| = \sqrt{\dfrac{16t^4 - 16t^2 + 4 + 16t^2 + 16t^2}{(2t^2+1)^4}} = \sqrt{\dfrac{16t^4 + 16t^2 + 4}{(2t^2+1)^4}} = \dfrac{2}{(2t^2+1)^2}\sqrt{4t^4+4t^2+1}$

$= \dfrac{2}{(2t^2+1)^2}\sqrt{(2t^2+1)^2} = \dfrac{2}{2t^2+1}$

$\hat{N}(t) = \left\langle \dfrac{-2t^2+1}{2t^2+1}, \dfrac{2t}{2t^2+1}, \dfrac{-2t}{2t^2+1} \right\rangle$

$\hat{T}(1) = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$

$\hat{N}(1) = \langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \rangle$

$$\hat{B}(1) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{vmatrix} = \hat{\imath}\left(-\frac{4}{9} - \frac{2}{9}\right) - \hat{\jmath}\left(-\frac{4}{9} + \frac{1}{9}\right) + \hat{k}\left(\frac{4}{9} + \frac{2}{9}\right)$$

$$= -\frac{2}{3}\hat{\imath} + \frac{1}{3}\hat{\jmath} + \frac{2}{3}\hat{k}$$

$\hat{B}(1) = \langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$

4pts

#2) $\vec{r}(t) = \langle 2t, \frac{1}{2}t^2, t^2 \rangle$

$\vec{r}\,'(t) = \langle 2, t, 2t \rangle \qquad \vec{r}\,''(t) = \langle 0, 1, 2 \rangle$

$K = \dfrac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3} \qquad \vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & t & 2t \\ 0 & 1 & 2 \end{vmatrix}$

$= \hat{\imath}(2t - 2t) - \hat{\jmath}(4) + \hat{k}(2)$

$|\vec{r}\,'(t) \times \vec{r}\,''(t)| = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$

$K = \dfrac{2\sqrt{5}}{(5t^2+4)^{3/2}} \qquad |\vec{r}\,'(t)| = \sqrt{4 + t^2 + 4t^2} = \sqrt{5t^2 + 4}$

3pts

#3.) $\vec{a}(t) = (5t)\hat{\imath} + (3t^2)\hat{\jmath} + (2t+3)\hat{k}$

$\vec{v}(t) = \left(\frac{5}{2}t^2\right)\hat{\imath} + (t^3)\hat{\jmath} + (t^2+3t)\hat{k} + \vec{C}_1 \qquad \vec{v}(0) = 6\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

$\vec{v}(0) = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k} + \vec{C}_1 = 6\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

$\vec{v}(t) = \left(\frac{5}{2}t^2 + 6\right)\hat{\imath} + (t^3 - 2)\hat{\jmath} + (t^2 + 3t + 5)\hat{k}$

$\vec{r}(t) = \left(\frac{5}{6}t^3 + 6t\right)\hat{\imath} + \left(\frac{1}{4}t^4 - 2t\right)\hat{\jmath} + \left(\frac{1}{3}t^3 + \frac{3}{2}t^2 + 5t\right)\hat{k} + \vec{C}_2 \qquad \vec{r}(0) = 4\hat{\imath} + 3\hat{\jmath}$

$\vec{r}(0) = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k} + \vec{C}_2 = 4\hat{\imath} + 3\hat{\jmath}$

$\vec{r}(t) = \left(\frac{5}{6}t^3 + 6t + 4\right)\hat{\imath} + \left(\frac{1}{4}t^4 - 2t + 3\right)\hat{\jmath} + \left(\frac{1}{3}t^3 + \frac{3}{2}t^2 + 5t\right)\hat{k}$

#4) $\vec{r}(t) = (80\sqrt{3})t\hat{\imath} + (96 + 80t - 16t^2)\hat{\jmath}$

3pts a.) $r_y = 96 + 80t - 16t^2$

$r_y' = 80 - 32t = 0$

$t = 80/32 = 2.5\text{ s}$

2pt. b.) $r_y = 96 + 80\left(\frac{5}{2}\right) - 16\left(\frac{5}{2}\right)^2$

$= 96 + 200 - 4(25) = 196\text{ ft}$

3pts. c.) $96 + 80t - 16t^2 = 0$

$-16(t^2 - 5t - 6) = 0$

$-16(t-6)(t+1) = 0$

$t = -1,\ t = 6$ 6 sec

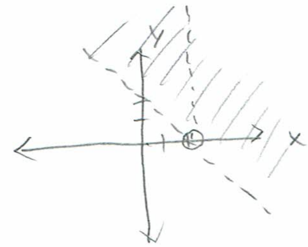
2pts. d.) $r_x = (80\sqrt{3})(6) = 480\sqrt{3}$ ft.

4pts
#5) $f(x,y) = \dfrac{\ln(x+y-2)}{x-2}$

$y > -x+2$

(0,0):
$0 > 2$ F

Domain: $\{(x,y) \mid x+y-2 > 0,\ x \neq 2\}$



4pts.
#6) $f(x,y) = \sqrt{x} + y$

$k=0$ $\quad \sqrt{x} + y = 0$

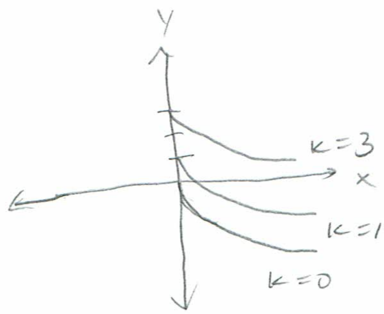
$y = -\sqrt{x}$

$k=1$ $\quad \sqrt{x} + y = 1$

$y = -\sqrt{x} + 1$

$k=3$ $\quad \sqrt{x} + y = 3$

$y = -\sqrt{x} + 3$



4pts
#7.) $\lim\limits_{(x,y)\to(2,0)} \dfrac{\sqrt{2x-y}-2}{2x-y-4} \to \dfrac{0}{0}$

$\lim\limits_{(x,y)\to(2,0)} \dfrac{\sqrt{2x-y}-2}{2x-y-4} \cdot \dfrac{\sqrt{2x-y}+2}{\sqrt{2x-y}+2} = \lim\limits_{(x,y)\to(2,0)} \dfrac{2x-y-4}{(2x-y-4)(\sqrt{2x-y}+2)} = \lim\limits_{(x,y)\to(2,0)} \dfrac{1}{\sqrt{2x-y}+2} = \dfrac{1}{4}$

4pts.
#8) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y}{x-y}$

x-axis ($y=0$)

$\lim\limits_{\substack{(x,y)\to(0,0)\\ y=0}} \dfrac{x^2-y}{x-y} = \lim\limits_{\substack{(x,y)\to(0,0)\\ y=0}} \dfrac{x^2}{x} = \lim\limits_{\substack{(x,y)\to(0,0)\\ y=0}} x = 0$

y-axis ($x=0$)

$\lim\limits_{\substack{(x,y)\to(0,0)\\ x=0}} \dfrac{x^2-y}{x-y} = \lim\limits_{\substack{(x,y)\to(0,0)\\ x=0}} \dfrac{-y}{-y} = \lim\limits_{\substack{(x,y)\to(0,0)\\ x=0}} 1 = 1$

Since $0 \neq 1$, then by the two paths approach, the limit does not exist.

#9.) $f(x,y) = x^2 \tan(xy)$

2pts. a.) $f_x = 2x\tan(xy) + x^2 y \sec^2(xy)$

2pts b.) $f_y = x^2 \cdot x\sec^2(xy) = x^3 \sec^2(xy)$

3pts. c.) $f_{xx} = 2\tan(xy) + 2xy\sec^2(xy) + 2xy\sec^2(xy) + x^2 y\, 2\sec(xy)\cdot y\sec(xy)\tan(xy)$

$= 2\tan(xy) + 4xy\sec^2(xy) + 2x^2y^2\sec^2(xy)\tan(xy)$

3pts d.) $f_{xy} = 2x \cdot x\sec^2(xy) + x^2\sec^2(xy) + x^2 y\, 2\sec(xy)\cdot x\sec(xy)\tan(xy)$

$= 2x^2\sec^2(xy) + x^2\sec^2(xy) + 2x^3y\sec^2(xy)\tan(xy)$

3pts e.) $f_{yy} = x^3 \cdot 2\sec(xy)\cdot x\,\sec(xy)\tan(xy) = 2x^4\sec^2(xy)\tan(xy)$

#10.) $w = 5x^3 + 7y^4z^5 \quad x = 3s^2 + 4t^5 \quad y = 7s^4 - 8t^2 \quad z = 6s^3 + 4t^4$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial z}$$

$$\frac{\partial w}{\partial s} = (15x^2)(6s) + (28y^3z^5)(28s^3) + (35y^4z^4)(18s^2)$$

$$= 90x^2s + 784y^3z^5s^3 + 630y^4z^4s^2$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$\frac{\partial w}{\partial t} = (15x^2)(20t^4) + (28y^3z^5)(-16t) + (35y^4z^4)(16t^3)$$

$$= 350x^2t^4 - 448y^3z^5t + 560y^4z^4t^3$$

4pts #11.) $4y^3 + 2\cos(x^5y^3) = 5x^3y^2 - 7x^6$

$F(x,y) = 4y^3 + 2\cos(x^5y^3) - 5x^3y^2 + 7x^6 = 0$

$$\frac{dy}{dx} = -\frac{-10x^4y^3\sin(x^5y^3) - 15x^2y^3 + 42x^5}{12y^2 - 6x^5y^2\sin(x^5y^3) - 10x^3y}$$

$F_x = -10x^4y^3\sin(x^5y^3) - 15x^2y^3 + 42x^5$

$F_y = 12y^2 - 6x^5y^2\sin(x^5y^3) - 10x^3y$

#12) $f(x,y) = x^3 e^{xy}$

4pts.
a.) $f_x = 3x^2 e^{xy} + x^3 y e^{xy}$ $\quad f_y = x^4 e^{xy}$

$f_x(1,0) = 3$ $\quad f_y(1,0) = 1$ $\quad f(1,0) = 1$

$z - 1 = 3(x-1) + 1(y-0)$

$z - 1 = 3x - 3 + y$ $\quad L(x,y) = 3x + y - 2$

$z = 3x + y - 2$

2pts.
b.) $L(1.3, 0.2) = 3(1.3) + 0.2 - 2 = 3.9 + 0.2 - 2 = 2.1$

4pts
#13.) $f(x,y) = \cos(xy) + e^{yz} + \ln(zx)$ $\quad P(1, 0, 1/2)$ $\quad \vec{u} = \hat{i} + 2\hat{j} + 2\hat{k}$

$|\vec{u}| = \sqrt{1+4+4} = 3$

$\vec{\nabla} f = \left(-y\sin(xy) + \frac{z}{zx}\right)\hat{i} + \left(-x\sin(xy) + ze^{yz}\right)\hat{j} + \left(ye^{yz} + \frac{x}{xz}\right)\hat{k}$ $\quad \hat{u} = \frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

$\vec{\nabla} f(1,0,1/2) = 1\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$ $\quad D_{\hat{u}} f = (1\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}) \cdot (\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k})$

$= \frac{1}{3} + \frac{1}{3} + \frac{4}{3} = 2$

#14) $x^2 + 2xy - y^2 + z^2 = 7$ $\quad P_0(1, -1, 3)$

4pts
a.) $f_x = (2x + 2y)$ $\quad f_y = (2x - 2y)$ $\quad f_z = 2z$

$f_x(1,-1,3) = 0$ $\quad f_y(1,-1,3) = 4$ $\quad f_z(1,-1,3) = 6$

$0(x-1) + 4(y+1) + 6(z-3) = 0$

$4y + 4 + 6z - 18 = 0$

$4y + 6z - 14 = 0$

$2y + 3z - 7 = 0$

2pts.
b)

$x = 1$

$y = -1 + 4t$

$z = 3 + 6t$

#15.) $f(x,y) = 4y\sqrt{x}$ $P_0(4,1)$

2pts
a.) $\vec{\nabla} f = 2yx^{-1/2}\hat{i} + 4x^{1/2}\hat{j}$

$\vec{\nabla} f(4,1) = \hat{i} + 8\hat{j}$

2pts
b.) $-\vec{\nabla} f = -\hat{i} - 8\hat{j}$

2pts
c.) $\vec{z}_1 = 8\hat{i} - \hat{j}$ $\vec{z}_2 = -8\hat{i} + \hat{j}$

6pts
#16) $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy$

$f_x = 12x - 6x^2 + 6y$ $f_y = 6y + 6x$

$12x - 6x^2 + 6y = 0$ AND $6y + 6x = 0$

$y = -x$

$12x - 6x^2 + 6(-x) = 0$

$-6x^2 + 6x = 0$

$-6x(x-1) = 0$

$x = 0$ or $x = 1$

$x=0$ $y=0$

$x=1$ $y=-1$

$f_{xx} = 12 - 12x$ $f_{xy} = 6$ $f_{yy} = 6$

$D(0,0) = (12)(6) - (6)^2 = 72 - 36 = 36 > 0$ $f_{xx} = 12 > 0$ (0,0) local min

$D(1,-1) = (0)(6) - (6)^2 = -36 < 0$ (1,-1) saddle point.