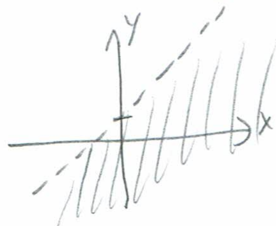
① $f(x,y) = \ln(x-y+1)$

$\{(x,y) \mid x-y+1 > 0\}$

4/



$y < x+1$

$f(x,y) = \ln(x+y-1)$

$\{(x,y) \mid x+y-1 > 0\}$

$y > -x+1$

②

4/

$f(x,y) = 4x^2 + 16y^2$

$k=0, k=4, k=16$

$4x^2 + 16y^2 = 0$

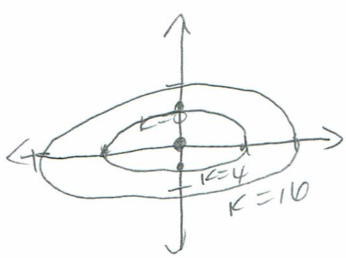
$4x^2 + 16y^2 = 4$

$x^2 + 4y^2 = 1$

$x^2 + \frac{y^2}{1/4} = 1$

$4x^2 + 16y^2 = 16$

$\frac{x^2}{4} + y^2 = 1$



$f(x,y) = 16x^2 + 4y^2$

$k=0, k=4, k=16$

$16x^2 + 4y^2 = 0$

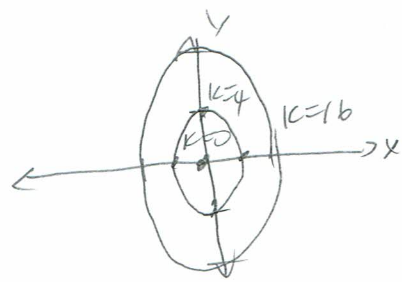
$16x^2 + 4y^2 = 4$

$4x^2 + y^2 = 1$

$\frac{x^2}{1/4} + y^2 = 1$

$16x^2 + 4y^2 = 16$

$x^2 + \frac{y^2}{4} = 1$



③

4/

$$\lim_{(x,y)\to(0,0)} \frac{x^8 - y^8}{x^4 - y^4}$$

$$= \lim_{(x,y)\to(0,0)} \frac{(x^4+y^4)\cancel{(x^4-y^4)}}{\cancel{x^4-y^4}} = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^8 - y^8}{x^4 + y^4}$$

$$= \lim_{(x,y)\to(0,0)} \frac{\cancel{(x^4+y^4)}(x^4-y^4)}{\cancel{(x^4+y^4)}} = 0$$

(4)

4

$$\lim_{(x,y)\to(0,0)} \frac{x+y-9}{\sqrt{x+y}-3} = 3$$

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt{x+y}-5}{x+y-25} = \frac{1}{5}$$

(5)

4

$$\lim_{(x,y)\to(0,0)} \frac{x^8-y^4}{x^8+y^4}$$

x-axis

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{x^8-y^4}{x^8+y^4} = \lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{x^8}{x^8} = 1$$

y-axis

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} \frac{x^8-y^4}{x^8+y^4} = \lim_{\substack{(x,y)\to(0,0)\\ x=0}} \left(-\frac{y^4}{y^4}\right) = -1$$

Since the limits are not equal, by the two paths approach, the limit DNE!

(6) $f(x,y) = 4x^2 e^{x^3y^2}$

2 a. $f_x = 8xe^{x^3y^2} + 4x^2 e^{x^3y^2}(3x^2y^2)$

$= 8xe^{x^3y^2} + 12x^4y^2 e^{x^3y^2}$

2 b. $f_y = 4x^2 e^{x^3y^2}(2x^3y)$

$= 8x^5 y e^{x^3y^2}$

3 c. $f_{xx} = 8e^{x^3y^2} + 8xe^{x^3y^2}(3x^2y^2)$

$+ 48x^3y^2 e^{x^3y^2} + 12x^4y^2 e^{x^3y^2}(3x^2y^2)$

$= 8e^{x^3y^2} + 72x^3y^2 e^{x^3y^2} + 36x^6y^4 e^{x^3y^2}$

3 d. $f_{xy} = 8xe^{x^3y^2}(2x^3y) + 24x^4y\, e^{x^3y^2}$

$+ 12x^4y^2 e^{x^3y^2}(2x^3y)$

$= 40x^4ye^{x^3y^2} + 24x^7y^3 e^{x^3y^2}$

3 e. $f_{yy} = 8x^5 e^{x^3y^2} + 8x^5 y e^{x^3y^2}(2x^3y)$

$= 8x^5 e^{x^3y^2} + 16x^8y^2 e^{x^3y^2}$

$f(x,y) = 3y^2 e^{x^2y^3}$

a. $f_x = 3y^2 e^{x^2y^3}(2xy^3)$

$= 6xy^5 e^{x^2y^3}$

b. $f_y = 6ye^{x^2y^3} + 3y^2 e^{x^2y^3}(3x^2y^2)$

$= 6ye^{x^2y^3} + 9x^2y^4 e^{x^2y^3}$

c. $f_{xx} = 6y^5 e^{x^2y^3} + 6xy^5 e^{x^2y^3}(2xy^3)$

$= 6y^5 e^{x^2y^3} + 12x^2y^8 e^{x^2y^3}$

d. $f_{xy} = 30xy^4 e^{x^2y^3} + 6xy^5 e^{x^2y^3}(3x^2y^2)$

$= 30xy^4 e^{x^2y^3} + 18x^3y^7 e^{x^2y^3}$

e. $f_{yy} = 6e^{x^2y^3} + 6ye^{x^2y^3}(3x^2y^2)$

$+ 36x^2y^3 e^{x^2y^3} + 9x^2y^4 e^{x^2y^3}(3x^2y^2)$

$= 6e^{x^2y^3} + 54x^2y^3 e^{x^2y^3} + 27x^4y^6 e^{x^2y^3}$

⑦ $w = x^2 + y^2 z,\quad x = 3s + t^2,\quad y = 4s^2 + 2t,\quad z = 5s - 3t$

4/

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

$$\frac{\partial w}{\partial s} = (2x)(3) + (2yz)(8s) + (y^2)(5)$$

$$= 6x + 16yzs + 5y^2$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$\frac{\partial w}{\partial t} = (2x)(2t) + (2yz)(2) + (y^2)(-3)$$

$$= 4xt + 4yz - 3y^2$$

$w = x^2 + yz^2,\quad x = 3t + s^2,\quad y = 5s + 4t^2,\quad z = 4s - 6t$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

$$\frac{\partial w}{\partial s} = (2x)(2s) + (z^2)(5) + (2yz)(4)$$

$$= 4xs + 5z^2 + 8yz$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$\frac{\partial w}{\partial t} = (2x)(3) + (z^2)(8t) + (2yz)(-6)$$

$$= 6x + 8z^2t - 12yz$$

⑧ $5x^3y^2 + \sin(x^4y^3) = 2y^4 + 3x^3$

4

$$5x^3y^2 + \sin(x^4y^3) - 2y^4 - 3x^3 = 0$$

$$F_x = 15x^2y^2 + 4x^3y^3\cos(x^4y^3) - 9x^2$$

$$F_y = 10x^3y + 3x^4y^2\cos(x^4y^3) - 8y^3$$

$$\frac{dy}{dx} = -\frac{15x^2y^2 + 4x^3y^3\cos(x^4y^3) - 9x^2}{10x^3y + 3x^4y^2\cos(x^4y^3) - 8y^3}$$

$3x^2y^4 + \sin(x^3y^2) = 4y^3 + 2x^5$

$$3x^2y^4 + \sin(x^3y^2) - 4y^3 - 2x^5 = 0$$

$$F_x = 6xy^4 + 3x^2y^2\cos(x^3y^2) - 10x^4$$

$$F_y = 12x^2y^3 + 2x^3y\cos(x^3y^2) - 12y^2$$

$$\frac{dy}{dx} = -\frac{6xy^4 + 3x^2y^2\cos(x^3y^2) - 10x^4}{12x^2y^3 + 2x^3y\cos(x^3y^2) - 12y^2}$$

⑨ $f(x,y) = \sqrt{xy} = (xy)^{1/2}$

$f_x = \frac{1}{2}(xy)^{-1/2}y \qquad f_y = \frac{1}{2}(xy)^{-1/2}x$

$f_x(1,4) = \frac{4}{2\sqrt{4}} = 1 \qquad f_y(1,4) = \frac{1}{2\sqrt{4}} = \frac{1}{4} \qquad f(1,4) = \sqrt{4} = 2$

4/ $L(x,y) = 2 + 1(x-1) + \frac{1}{4}(y-4) = 2 + x - 1 + \frac{1}{4}y - 1 = x + \frac{1}{4}y$

2/ $L(0.8, 4.3) = 0.8 + \frac{1}{4}(4.3) = 1.875$

(10)

4/

$f(x,y,z) = x^2y + y^2z + xz^2$

$\vec{\nabla} f = (2xy + z^2)\hat{i} + (2yz + x^2)\hat{j} + (y^2 + 2xz)\hat{k}$

$\vec{\nabla} f(2,1,3) = (4+9)\hat{i} + (6+4)\hat{j} + (1+12)\hat{k}$

$= 13\hat{i} + 10\hat{j} + 13\hat{k}$

$|\vec{v}| = \sqrt{1+4+4} = 3$

$\hat{v} = \langle -\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$\vec{\nabla} f \cdot \hat{v} = \langle 13, 10, 13 \rangle \cdot \langle -\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$= -\frac{13}{3} + \frac{20}{3} + \frac{26}{3}$

$= 11$

$f(x,y,z) = x^2y + y^2z + xz^2$

$\vec{\nabla} f = (2xy + z^2)\hat{i} + (2yz + x^2)\hat{j} + (y^2 + 2xz)\hat{k}$

$\vec{\nabla} f(3,1,2) = (6+4)\hat{i} + (4+9)\hat{j} + (1+12)\hat{k}$

$= 10\hat{i} + 13\hat{j} + 13\hat{k}$

$|\vec{v}| = \sqrt{4+4+1} = 3$

$\hat{v} = \langle \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \rangle$

$\vec{\nabla} f \cdot \hat{v} = \langle 10, 13, 13 \rangle \cdot \langle \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \rangle$

$= \frac{20}{3} + \frac{26}{3} - \frac{13}{3} = 11$

(11) $x^2y^2z^3 = 16$ $\quad P_0(2,2,1)$

$\vec{\nabla} f = (2xy^2z^3)\hat{i} + (2x^2yz^3)\hat{j} + (3x^2y^2z^2)\hat{k}$

$\vec{\nabla} f(2,2,1) = 16\hat{i} + 16\hat{j} + 48\hat{k}$

$16(x-2) + 16(y-2) + 48(z-1) = 0$

$16x - 32 + 16y - 32 + 48z - 48 = 0$

$x - 2 + y - 2 + 3z - 3 = 0$

5/ $\quad x + y + 3z - 7 = 0$

$x = 2 + 16t$

$y = 2 + 16t$

3/ $\quad z = 1 + 48t$

(12) $f(x,y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y$

$f_x = 6x^2 - 18x \qquad f_y = 6y^2 + 6y - 12$

7/

$6x(x-3) = 0 \qquad 6(y^2 + y - 2) = 0$

$6x(x-3) = 0 \qquad 6(y+2)(y-1) = 0$

$x = 0, \ x = 3 \qquad y = -2, \ y = 1$

$(0, -2)$ $\quad f_{xx} = 12x - 18$

$(0, 1)$ $\quad f_{xy} = 0$

$(3, -2)$ $\quad f_{yy} = 12y + 6$

$(3, 1)$

$D(0,-2) = (-18)(-18) > 0 \qquad f_{xx} = -18 < 0$ local max

$D(0,1) = (-18)(18) < 0$ saddle point

$D(3,-2) = (18)(-18) < 0$ saddle point

$D(3,1) = (18)(18) > 0 \qquad f_{xx} = 18 > 0$ local min

local max at $(0,-2)$

local min at $(3,1)$

saddle point at $(0,1)$, $(3,-2)$

(13) $f(x,y,z) = x + 2y + 3z$ $\qquad g(x,y,z) = x^2+y^2+z^2 = 25$

$\vec{\nabla} f = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ $\qquad \vec{\nabla} f = 2x\hat{\imath} + 2y\hat{\jmath} + 2z\hat{k}$

7/ $\quad 1 = 2x\lambda \quad 2 = 2y\lambda \quad 3 = 2z\lambda$

$x = \frac{1}{2\lambda} \quad y = \frac{2}{2\lambda} \quad z = \frac{3}{2\lambda}$

$\frac{1}{4\lambda^2} + \frac{4}{4\lambda^2} + \frac{9}{4\lambda^2} = 25$

$14 = 100\lambda^2$

$\pm \frac{\sqrt{14}}{10} = \lambda$

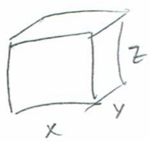
$\lambda = \frac{\sqrt{14}}{10}$

$x = \frac{5}{\sqrt{14}} \quad y = \frac{10}{\sqrt{14}} \quad z = \frac{15}{\sqrt{14}}$ max

$\lambda = -\frac{\sqrt{14}}{10}$

$x = -\frac{5}{\sqrt{14}} \quad y = -\frac{10}{\sqrt{14}} \quad z = -\frac{15}{\sqrt{14}}$ min

(14)



7/ $A = xy + 2xz + 2yz \qquad V = xyz = 8788$

$z = \frac{8788}{xy}$

$A = xy + 17576y^{-1} + 17576x^{-1}$

$A_x = y - 17576x^{-2} \qquad A_y = x - 17576y^{-2}$

$y - 17576x^{-2} = 0 \qquad x - 17576y^{-2} = 0$

$y = \frac{17576}{x^2} \qquad x = \frac{17576}{y^2}$

$y = 26 \qquad x = \frac{17576}{\left(\frac{17576}{x^2}\right)^2}$

$x = \frac{17576x^4}{17576^2}$

$z = 13 \qquad 17576 = x^3$

$x = 26$

26cm x 26cm x 13cm

$A = xy + 2xz + 2yz \qquad V = xyz = 5324$

$z = \frac{5324}{xy}$

$A = xy + 10648y^{-1} + 10648x^{-1}$

$A_x = y - 10648x^{-2} \qquad A_y = x - 10648y^{-2}$

$y - 10648x^{-2} = 0 \qquad x - 10648y^{-2} = 0$

$y = \frac{10648}{x^2} \qquad x = \frac{10648}{y^2}$

$y = 22 \qquad x = \frac{10648}{\left(\frac{10648}{x^2}\right)^2}$

$x = \frac{10648x^4}{10648^2}$

$10648 = x^3$

$z = 11 \qquad x = 22$

22cm x 22cm x 11cm