MATH 280 EXAM #2 KEY

#1 (4pts a.)

$\vec{r}(t) = \langle 6\sin(2t), 6\cos(2t), 5t \rangle$

$\vec{r}\,'(t) = \langle 12\cos(2t), -12\sin(2t), 5 \rangle$

$|\vec{r}\,'(t)| = \sqrt{144\cos^2(2t) + 144\sin^2(2t) + 25} = \sqrt{144+25} = \sqrt{169} = 13$

$\hat{T}(t) = \langle \frac{12}{13}\cos(2t), -\frac{12}{13}\sin(2t), \frac{5}{13} \rangle$

$\hat{T}'(t) = \langle -\frac{24}{13}\sin(2t), -\frac{24}{13}\cos(2t), 0 \rangle$

$|\hat{T}'(t)| = \sqrt{\left(\frac{24}{13}\sin 2t\right)^2 + \left(\frac{24}{13}\cos 2t\right)^2} = \frac{24}{13}$

$\hat{N}(t) = \langle -\sin(2t), -\cos(2t), 0 \rangle$

(4pts)

b) $\hat{B} = \hat{T} \times \hat{N} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{12}{13}\cos(2t) & -\frac{12}{13}\sin(2t) & \frac{5}{13} \\ -\sin(2t) & -\cos(2t) & 0 \end{vmatrix} = \hat{\imath}\left(\frac{5}{13}\cos(2t)\right) - \hat{\jmath}\left(\frac{5}{13}\sin(2t)\right)$

$+\hat{k}\left(-\frac{12}{13}\cos^2(2t) - \frac{12}{13}\sin^2(2t)\right)$

$= \hat{\imath}\left(\frac{5}{13}\cos(2t)\right) - \hat{\jmath}\left(\frac{5}{13}\sin(2t)\right) - \hat{k}\left(\frac{12}{13}\right)$

(4pts)

#2) $\vec{r}(t) = \langle \frac{4}{9}(1+t)^{3/2}, \frac{4}{9}(1-t)^{3/2}, \frac{1}{3}t \rangle$

$\vec{r}\,'(t) = \langle \frac{2}{3}(1+t)^{1/2}, -\frac{2}{3}(1-t)^{1/2}, \frac{1}{3} \rangle$ $\quad \vec{r}\,'(0) = \langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \rangle$

$\vec{r}\,''(t) = \langle \frac{1}{3}(1+t)^{-1/2}, \frac{1}{3}(1-t)^{-1/2}, 0 \rangle$ $\quad \vec{r}\,''(0) = \langle \frac{1}{3}, \frac{1}{3}, 0 \rangle$

$K = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$

$\vec{r}\,'(0) \times \vec{r}\,''(0) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 \end{vmatrix}$

$= \frac{\sqrt{\frac{1}{81} + \frac{1}{81} + \frac{16}{81}}}{\sqrt{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}}^{\,3}} = \frac{\sqrt{\frac{18}{81}}}{\sqrt{\frac{9}{9}}^{\,3}} = \frac{3\sqrt{2}}{9} = \frac{\sqrt{2}}{3}$

$= \hat{\imath}\left(-\frac{1}{9}\right) - \hat{\jmath}\left(-\frac{1}{9}\right) + \hat{k}\left(\frac{2}{9} + \frac{2}{9}\right)$

$= -\frac{1}{9}\hat{\imath} + \frac{1}{9}\hat{\jmath} + \frac{4}{9}\hat{k}$

(3pts)

#3) $\vec{a}(t) = (4t^2)\hat{i} + (6t+7)\hat{j} + (3t)\hat{k}$

$\vec{v}(t) = (\frac{4}{3}t^3)\hat{i} + (3t^2+7t)\hat{j} + (\frac{3}{2}t^2)\hat{k} + \vec{C}_1$

$\vec{v}(0) = \vec{C}_1 = 4\hat{i} + 3\hat{j} + 2\hat{k}$

$\vec{v}(t) = (\frac{4}{3}t^3+4)\hat{i} + (3t^2+7t+3)\hat{j} + (\frac{3}{2}t^2+2)\hat{k}$

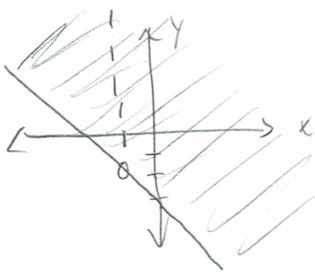
$\vec{r}(t) = (\frac{1}{3}t^4 + 4t)\hat{i} + (t^3 + \frac{7}{2}t^2 + 3t)\hat{j} + (\frac{1}{2}t^3 + 2t)\hat{k} + C_2$

$\vec{r}(0) = \vec{C}_2 = 2\hat{i} + 5\hat{j}$

$\vec{r}(t) = (\frac{1}{3}t^4 + 4t + 2)\hat{i} + (t^3 + \frac{7}{2}t^2 + 3t + 5)\hat{j} + (\frac{1}{2}t^3 + 2t)\hat{k}$

(4pts)

#4.) $f(x,y) = \dfrac{\sqrt{x+y+3}}{x+1}$ Domain $= \{(x,y) \mid x+y+3 \geq 0,\ x \neq -1\}$



$y \geq -x-3$

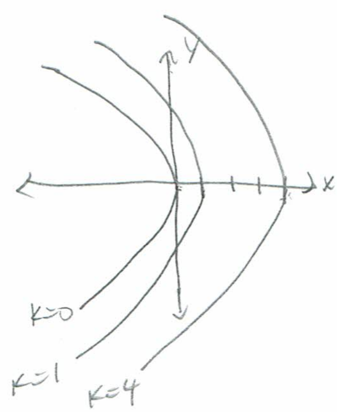
(4pts)

#5.) $f(x,y) = x+y^2$

$k=0 \quad x+y^2=0 \quad x=-y^2$

$k=1 \quad x+y^2=1 \quad x=1-y^2$

$k=4 \quad x+y^2=4 \quad x=4-y^2$



(4pts)

#6)

$$\lim_{(x,y)\to(0,0)} \frac{x^8-4y^4}{x^4+2y^2} = \lim_{(x,y)\to(0,0)} \frac{\cancel{(x^4+2y^2)}(x^4-2y^2)}{\cancel{x^4+2y^2}} = \lim_{(x,y)\to(0,0)} (x^4-2y^2) = 0$$

(4pts)
#7) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+2y^2}{x^2-y^2}$

x-axis: $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{x^2+2y^2}{x^2-y^2} = \lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{x^2}{x^2} = 1$

y-axis: $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ x=0}} \frac{x^2+2y^2}{x^2-y^2} = \lim_{\substack{(x,y)\to(0,0)\\ x=0}} \frac{2y^2}{-y^2} = -2$

Therefore, by the two-paths approach the limit does not exist.

#8.) $f(x,y) = y^2\cos(x^2y)$

(2pts) a.) $f_x = -y^2(2xy)\sin(x^2y) = -2xy^3\sin(x^2y)$

(3pts) b.) $f_y = 2y\cos(x^2y) - x^2y^2\sin(x^2y)$

(3pts) c.) $f_{xx} = -2y^3\sin(x^2y) - 4x^2y^4\cos(x^2y)$

(3pts) d.) $f_{xy} = -6xy^2\sin(x^2y) - 2x^3y^3\cos(x^2y)$

(3pts) e.) $f_{yy} = 2\cos(x^2y) - 2x^2y\sin(x^2y) - 2x^2y\sin(x^2y) - x^4y^2\cos(x^2y)$

$= 2\cos(x^2y) - 4x^2y\sin(x^2y) - x^4y^2\cos(x^2y)$

(4pts)
#9.) $\dfrac{\partial w}{\partial s} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial s} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial s}$

$= (6x)(12s) + (-20y^4z^2)(15s^2) + (-8y^5z)(10s^4) = 72xs - 300y^4z^2s^2 - 80y^5zs^4$

$\dfrac{\partial w}{\partial t} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial t}$

$= (6x)(8t^3) + (-20y^4z^2)(-9) + (-8y^5z)(12t^3) = 48xt^3 + 180y^4z^2 - 96y^5zt^3$

(4pts)
#10) $4x^3y^2 + 3\sin(x^4y^2) - 3y^4 + 7x^6 = 0$

$F_x = 12x^2y^2 + 12x^3y^2\cos(x^4y^2) + 42x^5$

$F_y = 8x^3y + 6x^4y\cos(x^4y^2) - 12y^3$

$\dfrac{dy}{dx} = -\dfrac{F_x}{F_y} = -\dfrac{12x^2y^2 + 12x^3y^2\cos(x^4y^2) + 42x^5}{8x^3y + 6x^4y\cos(x^4y^2) - 12y^3}$

#11.) $f(x,y) = x^3 e^{xy}$

(4pts) a.) $f_x = 3x^2 e^{xy} + y x^3 e^{xy}$ $\qquad$ $f_y = x^4 e^{xy}$

$f_x(1,0) = 3 + 0 = 3$ $\qquad$ $f_y(1,0) = 1$ $\qquad$ $f(1,0) = 1$

$$z - 1 = 3(x-1) + 1(y-0)$$
$$z - 1 = 3x - 3 + y$$
$$z = 3x + y - 2$$
$$L(x,y) = 3x + y - 2$$

(2pts) b.) $L(1.2, 0.3) = 3(1.2) + 0.3 - 2 = 1.9$

(4pts)
#12.) $f(x,y,z) = x^3 - xy^2 - z$ $\qquad$ $\vec{u} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$|\vec{u}| = \sqrt{4+9+36} = 7$

$\vec{\nabla} f = (3x^2 - y^2)\hat{i} + (-2xy)\hat{j} + (-1)\hat{k}$

$\vec{\nabla} f(1,1,0) = 2\hat{i} - 2\hat{j} - 1\hat{k}$ $\qquad$ $\hat{u} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

$\vec{\nabla} f \cdot \hat{u} = (2\hat{i} - 2\hat{j} - 1\hat{k}) \cdot (\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}) = \frac{4}{7} + \frac{6}{7} - \frac{6}{7} = \frac{4}{7}$

#13) $f(x,y,z) = x^2 + y^2 - z^2 = 18$

(4pts) a.) $\vec{\nabla} f = 2x\hat{i} + 2y\hat{j} - 2z\hat{k}$

$\vec{\nabla} f(3,5,-4) = 6\hat{i} + 10\hat{j} + 8\hat{k}$

$$6(x-3) + 10(y-5) + 8(z+4) = 0$$
$$6x - 18 + 10y - 50 + 8z + 32 = 0$$
$$6x + 10y + 8z - 36 = 0$$
$$3x + 5y + 4z - 18 = 0$$

(2pts) b.)
$$x = 3 + 6t$$
$$y = 5 + 10t$$
$$z = -4 + 8t$$

#14) $f(x,y,z) = x^2+xy+y^2 \quad P(-2,1)$

(2pts) a.) $\vec{\nabla}f = (2x+y)\hat{i} + (x+2y)\hat{j}$

$\vec{\nabla}f = -3\hat{i} + 0\hat{j}$

(2pts) b.) $-\vec{\nabla}f = 3\hat{i}$

(2pts) c.) $\vec{z}_1 = 3\hat{j}$

$\vec{z}_2 = -3\hat{j}$

(6pts)
#15.) $f(x,y) = 6x^2-2x^3+3y^2+6xy$

$f_x = 12x - 6x^2+6y \qquad f_y = 6y+6x$

$12x-6x^2+6y=0 \qquad 6y+6x=0$

$12x-6x^2-6x=0 \qquad y=-x$

$-6x^2+6x=0$

$-6x(x-1)=0$

$x=0,1 \qquad x=0 \ \ y=0$

$x=1 \ \ y=-1$

$f_{xx} = 12-12x \qquad f_{xy} = 6 \qquad f_{yy} = 6$

$D(0,0) = (12)(6) - 6^2 = 36 > 0 \qquad f_{xx} = 12 > 0$ local min

$D(1,-1) = (0)(6) - 6^2 = -36 < 0$ saddle point

(0,0) local min

(1,-1) saddle point