

⋮ **Question #1** Pick 1 questions, 0 pts per question



⋮ **Question**

(4 points) 1. For the following function, determine and sketch the domain:  $f(x, y) = \sqrt{y + x + 1}$

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⋮ **Question #2** Pick 1 questions, 0 pts per question



⋮ **Question**

(4 points) 2. Given  $f(x, y) = x^2 + y$ . Sketch the function's level curves for  $k = 0$ ,  $k = 1$ ,  $k = 3$ .



⋮ Question #3 Pick 1 questions, 0 pts per question



⋮ Question

(4 points) 3. Find the following limit:  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^4}{x^2 - y^2}$

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⋮ Question #4 Pick 1 questions, 0 pts per question



⋮ Question

(4 points) 4. Find the following limit:  $\lim_{(x,y) \rightarrow (3,6)} \frac{x+y-9}{\sqrt{x+y}-3}$

⋮ Question

(4 points) 4. Find the following limit:  $\lim_{(x,y) \rightarrow (16,9)} \frac{x+y-25}{\sqrt{x+y}-5}$

⋮ Question #5 Pick 1 questions, 0 pts per question





⋮ Question

(4 points) 5. Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^4}{x^4 + y^4}$  does not exist by using the two-paths approach.

⋮ Question #6 Pick 1 questions, 0 pts per question



⋮ Question

6. Given  $f(x, y) = 4x^3 \sin(x^4 y^2)$ .

(2 points) a. Find  $f_x$

(2 points) b. Find  $f_y$

(3 points) c. Find  $f_{xx}$

(3 points) d. Find  $f_{xy}$

(3 points) e. Find  $f_{yy}$

⋮ Question

6. Given  $f(x, y) = 4y^2 \cos(x^3 y^4)$ .

(2 points) a. Find  $f_x$

(2 points) b. Find  $f_y$

(3 points) c. Find  $f_{xx}$

(3 points) d. Find  $f_{xy}$

(3 points) e. Find  $f_{yy}$



⋮ **Question #7** Pick 1 questions, 0 pts per question



⋮ Question

(4 points) 7. Given  $z = x^3 + y^2$ ,  $x = 3s^4 + 5t^3$ ,  $y = 6s^2 + 7t^5$ . Find  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$ .

⋮ Question

(4 points) 7. Given  $z = x^4 + y^3$ ,  $x = 5s^3 + 4t^2$ ,  $y = 3s^5 + 6t^4$ . Find  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$ .

⋮ **Question #8** Pick 1 questions, 0 pts per question



⋮ Question

(4 points) 8. Use partial derivatives to perform implicit differentiation to find  $\frac{dy}{dx}$  if  $4y^3 + e^{x^3y^2} = 5x^4y^3 + 6x^2$ , where y is a differentiable function of x.

⋮ Question



(4 points) 8. Use partial derivatives to perform implicit differentiation to find  $\frac{dy}{dx}$  if  $3x^4 + e^{x^3y^2} = 4x^3y^5 + 7x^3$ , where  $y$  is a differentiable function of  $x$ .

⋮ Question #9 Pick 1 questions, 0 pts per question



⋮ Question

9. Given  $f(x, y) = y^2 e^{xy}$ .

(4 points) a. Find the linearization,  $L(x, y)$ , of the function at  $(0, 2)$ .

(2 points) b. Use the linearization to approximate the function at the point  $(0.2, 2.1)$ .

⋮ Question #10 Pick 1 questions, 0 pts per question



⋮ Question

(4 points) 10. Find the directional derivative of

$f(x, y, z) = x^2y + x\sqrt{1+z}$  at the point  $(1, 2, 3)$  in the direction of the vector  $\vec{v} = \langle 2, 1, -2 \rangle$ .

⋮ Question #11 Pick 1 questions, 0 pts per question





⋮ Question

11. Given  $xy^2z^3 = 8$  and the point  $P_0(2, 2, 1)$ .

(5 points) a. Find the equation of the tangent line at the point  $P_0$ .

(2 points) b. Find the equation of the normal line to the given surface at the point  $P_0$ .

⋮ Question #12 Pick 1 questions, 0 pts per question



⋮ Question

12. Given  $f(x, y) = x^2y + \sqrt{y}$  and the point  $P_0(2, 1)$

(2 points) a. Find the direction of most rapid increase at  $P_0$

(2 points) b. Find the direction of most rapid decrease at  $P_0$ .

(2 points) c. Find the directions of zero change at  $P_0$ .

⋮ Question #13 Pick 1 questions, 0 pts per question



⋮ Question

(7 points) 13. Given  $f(x, y) = x^3 - 6xy + 8y^3$ . Find the local maxima, local minima, and saddle points.

⋮ Question #14 Pick 1 questions, 0 pts per question



⋮ Question

(7 points) 14. Use Lagrange multipliers to determine the maximum and minimum values of  $f(x, y, z) = xyz$  subject to the constraint  $g(x, y, z) = x^2 + y^2 + z^2 = 3$ .

⋮ Question #15 Pick 1 questions, 0 pts per question



⋮ Question

(7 points) 15. Find the dimensions of a rectangular box with the largest volume if the total surface area is given as  $64 \text{ cm}^2$ .

⋮ Question 16 Pick 1 questions, 0 pts per question



⋮ Question

(4 points extra credit) 16. Use the definition of the directional derivative to find the derivative of  $f(x, y) = x^2 - 4y^2$  at the point  $P_0(2, -1)$  in the direction  $\hat{u} = \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$ .