

MATH 280 - QUIZ #1

Name: 149

Directions: Please show all work to receive maximum credit. This quiz is worth 14 points.

1. Given $\bar{a} = \langle 2, 4, -3 \rangle$ and $\bar{b} = \langle 1, -3, 2 \rangle$. Determine the following:

$$(2 \text{ points}) \text{ a. } \bar{a} \cdot \bar{b} = 2 - 12 - 6 = -16$$

$$\begin{aligned} (1 \text{ point}) \text{ b. } 5\bar{a} + 3\bar{b} &= 5\langle 2, 4, -3 \rangle + 3\langle 1, -3, 2 \rangle \\ &= \langle 10, 20, -15 \rangle + \langle 3, -9, 6 \rangle \\ &= \langle 13, 11, -9 \rangle \end{aligned}$$

$$\begin{aligned} (2 \text{ points}) \text{ c. } \text{proj}_{\bar{a}} \bar{b} &= \frac{\bar{a} \cdot \bar{b}}{\|\bar{a}\|^2} \langle 2, 4, -3 \rangle \quad \|\bar{a}\| = \sqrt{4+16+9} = \sqrt{29} \\ &= \frac{-16}{29} \langle 2, 4, -3 \rangle \\ &= \left\langle \frac{-32}{29}, \frac{-64}{29}, \frac{48}{29} \right\rangle \end{aligned}$$

- (2 point) d. The area of the parallelogram determined by \bar{a} and \bar{b} .

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & -3 \\ 1 & -3 & 2 \end{vmatrix} = \hat{i}(8-9) - \hat{j}(4+3) + \hat{k}(-6-4) \\ = -\hat{i} - 7\hat{j} - 10\hat{k}$$

$$|\bar{a} \times \bar{b}| = \sqrt{1+49+100} = \sqrt{150} = \sqrt{25 \cdot 6} = 5\sqrt{6} \text{ sq units}$$

(3 points) 2. Find the parametric form of the equation of a line that passes through the points $P(4,8,-7)$ and $Q(-1,5,6)$.

$$\vec{PQ} = \langle -5, -3, 13 \rangle$$

$$x = 4 - 5t$$

$$y = 8 - 3t$$

$$z = -7 + 13t$$

(4 points) 3. Find the equations of the plane that passes through the points $P(3,2,-5)$, $Q(4,3,6)$, and $R(1,-3,2)$.

$$\vec{PQ} = \langle 1, 1, 11 \rangle$$

$$\vec{PR} = \langle -2, -5, 7 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 11 \\ -2 & -5 & 7 \end{vmatrix} = \hat{i}(7+55) - \hat{j}(-7+22) + \hat{k}(-5+2) \\ = 62\hat{i} - 35\hat{j} - 3\hat{k}$$

$$62(x-3) - 35(y-2) - 3(z+5) = 0$$

$$62x - 186 - 35y + 70 - 3z - 15 = 0$$

$$62x - 35y - 3z - 143 = 0$$