MATH 280 – QUIZ #3 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 14 points. Good luck!

(3 points) 1. Given $w = x^2 + yz$, $x = 3t^2 + 1$, $y = 2t - 4$, $z = t^3$. Find $\frac{dw}{dt}$.

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$\frac{dw}{dt} = (2x)(3t) + (z)(2) + (y)(3t^2)$$

$$= 6t(3t^2+1) + 2(t^3) + 3t^2(2t-4)$$

$$= 18t^3 + 6t + 2t^3 + 6t^3 - 12t^2$$

$$= 26t^3 - 12t^2 + 6t$$

(3 points) 2. Find the derivative of the function $f(x, y) = 2xy - 3y^2$ at $P_0(5, 5)$ in the direction of $\vec{A} = 4\hat{i} + 3\hat{j}$.

$$|\vec{A}| = \sqrt{4^2 + 3^2} = 5$$

$$\hat{u} = \frac{\vec{A}}{|\vec{A}|} = \frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$$

$$\vec{\nabla} f = (2y)\hat{i} + (2x - 6y)\hat{j}$$

$$\vec{\nabla} f(5,5) = 10\hat{i} + (-20)\hat{j}$$

$$D_{\hat{u}} f = \vec{\nabla} f|_{P_0} \cdot \hat{u} = (10\hat{i} - 20\hat{j}) \cdot \left(\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right)$$

$$= 8 - 12 = -4$$

(3 points) *3.* Find the equation of the tangent plane and the equation of the normal line at the point $P_0(3,5,-4)$ on the surface $x^2+y^2-z^2=18$.

$f(x,y,z)=x^2+y^2-z^2=18$

$\vec{\nabla} f = 2x\hat{\imath} + 2y\hat{\jmath} - 2z\hat{k}$

$\vec{\nabla} f|_{P_0} = 6\hat{\imath} + 10\hat{\jmath} + 8\hat{k}$

Tangent Plane

$6(x-3)+10(y-5)+8(z+4)=0$

$6x-18+10y-50+8z+32=0$

$6x+10y+8z-36=0$

Normal Line

$x=3+6t$

$y=5+10t$

$z=-4+8t$

(5 points) *4.* Find all the local maxima, local minima, and saddle points of the given function.

$$f(x)=6x^2-2x^3+3y^2+6xy$$

$f_x = 12x - 6x^2 + 6y$ $\qquad$ $f_y = 6y + 6x$

$12x-6x^2+6y=0$ $\qquad$ $6y+6x=0$

$12x-6x^2-6x=0$ $\qquad$ $y=-x$

$-6x^2+6x=0$

$-6x(x-1)=0$

$x=0,1$ $\qquad$ $x=0 \quad y=0$

$x=1, \quad y=-1$

$f_{xx}=12-12x$ $\qquad$ $f_{yy}=6$ $\qquad$ $f_{xy}=6$

$D = f_{xx}f_{yy}-(f_{xy})^2 = (12-12x)(6)-(6)^2$

$(0,0)$: $(12-12(0))(6)-(6)^2 = 72-36 = 36>0$ $\qquad$ $f_{xx}=12>0$ local min

$(1,-1)$ $\quad$ $(12-12(1))(6)-(6)^2 = 0-36 = -36<0$ $\qquad$ saddle point

$(0,0)$ local minimum $\qquad$ $(1,-1)$ saddle point