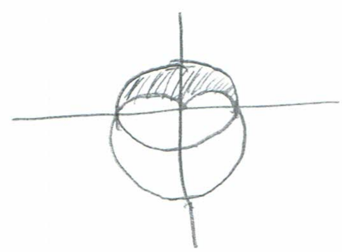
MATH 280 – QUIZ #4 *Name:* Key

Directions: Please show all work for maximum credit. This quiz is worth 14 points. There are 15 points on this quiz. Good luck!

(6 points) 1. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \sin\theta$.

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	1	0	1	2	0



$1 - \sin\theta = 1$

$\sin\theta = 0$

$\theta = 0, \pi$

$$\int_0^{\pi}\int_{1-\sin\theta}^{1} r\,dr\,d\theta = \int_0^{\pi} \frac{r^2}{2}\Big|_{1-\sin\theta}^{1} = \frac{1}{2}\int_0^{\pi}\left[1^2 - (1-\sin\theta)^2\right]d\theta$$

$$= \frac{1}{2}\int_0^{\pi}\left[1 - (1 - 2\sin\theta + \sin^2\theta)\right]d\theta = \frac{1}{2}\int_0^{\pi}(2\sin\theta - \sin^2\theta)\,d\theta$$

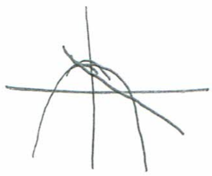
$$= \frac{1}{2}\int_0^{\pi}\left[2\sin\theta - \left(\frac{1-\cos 2\theta}{2}\right)\right]d\theta = \frac{1}{2}\int_0^{\pi}\left(2\sin\theta - \frac{1}{2} + \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= \frac{1}{2}\left[-2\cos\theta - \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]\Big|_0^{\pi}$$

$$= \frac{1}{2}\left[\left(2 - \frac{1}{2}\pi\right) - (-2)\right] = \frac{1}{2}\left[4 - \frac{\pi}{2}\right] = 2 - \frac{\pi}{4}$$

(6 points) 2. Evaluate the double integral $\iint_D (1-x+2y)\,dA$ where D is the region bounded by $x+y=1$ and $x^2+y=1$.

$y=1-x \quad y=1-x^2$



$1-x^2=1-x$

$0=x^2-x$

$x(x-1)=0$

$x=0,1$

$$\int_0^1\int_{1-x}^{1-x^2}(1-x+2y)\,dy\,dx=\int_0^1 (y-xy+y^2)\Big|_{1-x}^{1-x^2}dx$$

$$=\int_0^1\Big[[(1-x^2)-x(1-x^2)+(1-x^2)^2]-[(1-x)-x(1-x)+(1-x)^2]\Big]dx$$

$$=\int_0^1[(1-x^2-x+x^3+1-2x^2+x^4)-(1-x-x+x^2+1-2x+x^2)]\,dx$$

$$=\int_0^1(1-x-3x^2+x^3+x^4)-(1-4x+x^2)]\,dx$$

$$=\int_0^1(3x-5x^2+x^3+x^4)\,dx=\frac{3x^2}{2}-\frac{5x^3}{3}+\frac{x^4}{4}+\frac{x^5}{5}\Big|_0^1$$

$$=\frac{3}{2}-\frac{5}{3}+\frac{1}{4}+\frac{1}{5}=\frac{90-100+15+12}{60}=\frac{17}{60}$$

(3 points) 3. Evaluate the following integral: $\int_0^1\int_0^{2-2x}\int_0^{2-2x-y}dz\,dy\,dx$

$$=\int_0^1\int_0^{2-2x}z\Big|_0^{2-2x-y}dy\,dx=\int_0^1\int_0^{2-2x}(2-2x-y)\,dy\,dx$$

$$=\int_0^1\left(2y-2xy-\frac{y^2}{2}\right)\Big|_0^{2-2x}dx=\int_0^1\left(2(2-2x)-2x(2-2x)-\frac{(2-2x)^2}{2}\right)dx$$

$$=\int_0^1\left(4-4x-4x+4x^2-\left(\frac{4-8x+4x^2}{2}\right)\right)dx=\int_0^1(4-8x+4x^2-2+4x-2x^2)\,dx$$

$$=\int_0^1(2-4x+2x^2)\,dx=\left(2x-2x^2+\frac{2x^3}{3}\right)\Big|_0^1=2-2+\frac{2}{3}=\frac{2}{3}$$