MATH 280 - Quiz #1

(3pts) #1) $P(3,-5,1)\ Q(6,2,-2)$

$$d=\sqrt{(6-3)^2+(2+5)^2+(-2-1)^2}=\sqrt{3^2+7^2+(-3)^2}=\sqrt{9+49+9}=\sqrt{67}$$

(3pts) #2) $x^2+y^2+z^2-8x+2y+6z+1=0$

$$x^2-8x+16+y^2+2y+1+z^2+6z+9=-1+16+1+9$$

$$(x-4)^2+(y+1)^2+(z+3)^2=25$$

center $(4,-1,-3)$ radius $=5$

#3) $\vec{a}=\langle -3,6,2\rangle \quad \vec{b}=\langle 5,4,7\rangle$

(2pts) a) $5\vec{a}-3\vec{b}=5\langle -3,6,2\rangle-3\langle 5,4,7\rangle=\langle -15,30,10\rangle+\langle -15,-12,-21\rangle=\langle -30,18,-11\rangle$

(2pts) b) $\vec{a}\cdot\vec{b}=\langle -3,6,2\rangle\cdot\langle 5,4,7\rangle=-15+24+14=23$

(3pts) c) $\text{proj}_{\vec{b}}\vec{a}=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\dfrac{\vec{b}}{|\vec{b}|}=\dfrac{23}{\sqrt{90}}\dfrac{\langle 5,4,7\rangle}{\sqrt{90}}=\dfrac{23}{90}\langle 5,4,7\rangle=\left\langle \dfrac{115}{90},\dfrac{92}{90},\dfrac{161}{90}\right\rangle=\left\langle \dfrac{115}{90},\dfrac{46}{45},\dfrac{161}{90}\right\rangle$

$$|\vec{b}|=\sqrt{5^2+4^2+7^2}=\sqrt{25+16+49}=\sqrt{90}$$

(4pts) d) $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-3&6&2\\5&4&7\end{vmatrix}=\hat{i}(42-8)-\hat{j}(-21-10)+\hat{k}(-12-30)$

$$=34\hat{i}+31\hat{j}-42\hat{k}$$

(3pts) e) $|\vec{a}\times\vec{b}|=\sqrt{34^2+31^2+(-42)^2}=\sqrt{3881}$

(4pts) #4) $W=\vec{F}\cdot\vec{D}=|\vec{F}||\vec{D}|\cos\theta$

$$=(35\text{lb})(85\text{ft})\cos 40^\circ$$

$$=2279.0\ \text{ft-lb}$$