

MATH 280 – QUIZ #2

Name: Key

**Directions:** This is a take home quiz. This quiz is due at the beginning of class on Monday, October 14, 2019. Please show all work for maximum credit. This quiz is worth 14 points. Good luck!

1. Given the following position function:  $\vec{r}(t) = (3 \sin t)\hat{i} + (3 \cos t)\hat{j} + (4t)\hat{k}$ . Find the following.

(4 points) a.  $\hat{N}$

$$\vec{v}'(t) = (3 \cos t)\hat{i} + (-3 \sin t)\hat{j} + 4\hat{k}$$

$$|\vec{v}'(t)| = \sqrt{(3 \cos t)^2 + (-3 \sin t)^2 + (4)^2} = \sqrt{9 \cos^2 t + 9 \sin^2 t + 16} = \sqrt{9 + 16} = 5$$

$$\hat{T} = \frac{\vec{v}'(t)}{|\vec{v}'(t)|} = \left(\frac{3}{5} \cos t\right)\hat{i} + \left(-\frac{3}{5} \sin t\right)\hat{j} + \frac{4}{5}\hat{k}$$

$$\hat{T}' = \left(-\frac{3}{5} \sin t\right)\hat{i} + \left(-\frac{3}{5} \cos t\right)\hat{j}$$

$$|\hat{T}'| = \sqrt{\left(-\frac{3}{5} \sin t\right)^2 + \left(-\frac{3}{5} \cos t\right)^2} = \sqrt{\frac{9}{25} \sin^2 t + \frac{9}{25} \cos^2 t} = \frac{3}{5}$$

$$\hat{N} = \frac{\hat{T}'}{|\hat{T}'|} = (-\sin t)\hat{i} + (-\cos t)\hat{j}$$

(4 points) b.  $\hat{B} = \hat{T} \times \hat{N}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{5} \cos t & -\frac{3}{5} \sin t & \frac{4}{5} \\ -\sin t & -\cos t & 0 \end{vmatrix}$$

$$= \hat{i} \left(\frac{4}{5} \cos t\right) - \hat{j} \left(\frac{4}{5} \sin t\right) + \hat{k} \left(\frac{3}{5} \cos^2 t - \frac{3}{5} \sin^2 t\right)$$

$$= \left(\frac{4}{5} \cos t\right)\hat{i} + \left(-\frac{4}{5} \sin t\right)\hat{j} + \left(-\frac{3}{5}\right)\hat{k}$$

(2 points) 2. Find the following limit:  $\lim_{\substack{(x,y) \rightarrow (3,1) \\ x \neq 3}} \frac{xy - 3y - 3x + 9}{x - 3}$

$$= \lim_{\substack{(x,y) \rightarrow (3,1) \\ x \neq 3}} \frac{y(x-3) - 3(x-3)}{x-3} = \lim_{\substack{(x,y) \rightarrow (3,1) \\ x \neq 3}} \frac{(x-3)(y-3)}{x-3} = \lim_{\substack{(x,y) \rightarrow (3,1) \\ x \neq 3}} y-3 = 1-3 = -2$$

(2 points) 3. Show that  $f(x,y) = \frac{x^8}{x^8 - y^2}$  has no limit as  $(x,y) \rightarrow (0,0)$  by considering different paths of approach. Use  $y = kx^4$ ,  $k \neq 1$ , and explain the result.

$$\lim_{\substack{(x,y) \rightarrow (0,0) \\ y = kx^4, k \neq 1}} \frac{x^8}{x^8 - y^2} = \lim_{(x,y) \rightarrow (0,0)} \frac{x^8}{x^8 - (kx^4)^2} = \lim_{(x,y) \rightarrow (0,0)} \frac{x^8}{x^8 - k^2 x^8} = \frac{1}{1 - k^2}$$

Since the limit depends only on  $k$ , the limit does not exist.

or if  $k=2$ , limit =  $-\frac{1}{3}$   
 if  $k=3$ , limit =  $-\frac{1}{8}$   
 since the limits are different given different paths, the limit does not exist.

(3 points) 4. Given the following function:  $f(x,y) = x^2 + y^2$ . Sketch the function's level curves when  $k = 0$ ,  $k = 9$ , and  $k = 25$ .

$k=0 \quad x^2 + y^2 = 0$

$k=9 \quad x^2 + y^2 = 9$

$k=25 \quad x^2 + y^2 = 25$

