MATH 280 – QUIZ #2 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

1. Given the following position function: $\vec{r}(t) = (6\sin 2t)\hat{i} + (6\cos 2t)\hat{j} + 5t\hat{k}$. Find the following.

(4 points) a. $\hat{N}$

$\vec{r}'(t) = (12\cos 2t)\hat{i} + (-12\sin 2t)\hat{j} + 5\hat{k}$

$|\vec{r}'(t)| = \sqrt{144\cos^2 2t + 144\sin^2 2t + 25} = 13$

$\hat{T} = \left(\frac{12}{13}\cos 2t\right)\hat{i} + \left(-\frac{12}{13}\sin 2t\right)\hat{j} + \frac{5}{13}\hat{k}$

$\hat{T}' = \left(-\frac{24}{13}\sin 2t\right)\hat{i} + \left(-\frac{24}{13}\cos 2t\right)\hat{j}$

$|\hat{T}'| = \sqrt{\frac{576}{169}\sin^2 2t + \frac{576}{169}\cos^2 2t} = \sqrt{\frac{576}{169}} = \frac{24}{13}$

$\hat{N} = -\sin 2t\,\hat{i} - \cos 2t\,\hat{j}$

(4 points) b. $\hat{B}$

$$\hat{T} \times \hat{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{12}{13}\cos 2t & -\frac{12}{13}\sin 2t & \frac{5}{13} \\ -\sin 2t & -\cos 2t & 0 \end{vmatrix}$$

$= \hat{i}\left(\frac{5}{13}\cos 2t\right) - \hat{j}\left(\frac{5}{13}\sin 2t\right) + \hat{k}\left(-\frac{12}{13}\cos^2 2t - \frac{12}{13}\sin^2 2t\right)$

$= \frac{5}{13}\cos 2t\,\hat{i} - \frac{5}{13}\sin 2t\,\hat{j} - \frac{12}{13}\hat{k}$

(3 points) 2. Find the following limit: $\lim\limits_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \dfrac{x+y-4}{\sqrt{x+y}-2}$ $\to \frac{0}{0}$

$$= \lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}\cdot\frac{\sqrt{x+y}+2}{\sqrt{x+y}+2} = \lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{(x+y-4)(\sqrt{x+y}+2)}{(x+y-4)}$$

$$= \lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} (\sqrt{x+y}+2) = \sqrt{2+2}+2 = \sqrt{4}+2 = 4$$

(3 points) 3. Show that $f(x,y) = \dfrac{x^6}{x^6 - y^2}$ has no limit as $(x,y)\to(0,0)$ by considering different paths of approach. Use $y = kx^3$ and <u>explain</u> the result.

$$\frac{x^6}{x^6-(kx^3)^2} = \frac{x^6}{x^6-k^2x^6} = \frac{x^6}{x^6-k^2x^6} = \frac{x^6}{x^6(1-k^2)} = \frac{1}{1-k^2}$$

The limit will depend on k (for $k\neq 1$). So, if $k=0$, limit $=1$ and if $k=2$, limit $=-1/3$. $\therefore$ The limit does not exist by the two paths approach.

(3 points) 4. Given the following function: $f(x,y) = x^2 + y^2$. Sketch the function's level curves when $k = 0$, $k = 9$, and $k = 25$.

$k=0$; $x^2+y^2=0$ point

$k=9$ $x^2+y^2=9$ circle of radius 3

$k=25$ $x^2+y^2=25$ circle of radius 5