MATH 280 – QUIZ #2 *Name:* KEY

Directions: This is a take home quiz. This quiz is due at the beginning of class on Thursday, October 10, 2019. Please show all work for maximum credit. This quiz is worth 14 points. Good luck!

1. Given the following position function: $\vec{r}(t) = (4\cos t)\hat{i} + (3t)\hat{j} + (4\sin t)\hat{k}$. Find the following.

(4 points) a. $\hat{N}$

$\vec{r}\,'(t) = (-4\sin t)\hat{i} + 3\hat{j} + (4\cos t)\hat{k}$

$|\vec{r}\,'(t)| = \sqrt{(-4\sin t)^2 + (3)^2 + (4\cos t)^2}$

$= \sqrt{16\sin^2 t + 9 + 16\cos^2 t} = \sqrt{16+9} = 5$

$\hat{T} = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \left(-\frac{4}{5}\sin t\right)\hat{i} + \frac{3}{5}\hat{j} + \left(\frac{4}{5}\cos t\right)\hat{k}$

$\hat{T}' = \left(-\frac{4}{5}\cos t\right)\hat{i} + \left(-\frac{4}{5}\sin t\right)\hat{k}$

$|\hat{T}'| = \sqrt{\left(-\frac{4}{5}\cos t\right)^2 + \left(-\frac{4}{5}\sin t\right)^2} = \sqrt{\frac{16}{25}\cos^2 t + \frac{16}{25}\sin^2 t} = \frac{4}{5}$

$\hat{N} = \dfrac{\hat{T}'}{|\hat{T}'|} = (-\cos t)\hat{i} + (-\sin t)\hat{k}$

(4 points) b. $\hat{B}$ $= \hat{T} \times \hat{N}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{4}{5}\sin t & \frac{3}{5} & \frac{4}{5}\cos t \\ -\cos t & 0 & -\sin t \end{vmatrix}$$

$= \hat{i}\left(-\frac{3}{5}\sin t\right) - \hat{j}\left(\frac{4}{5}\sin^2 t + \frac{4}{5}\cos^2 t\right) + \hat{k}\left(\frac{3}{5}\cos t\right)$

$= \left(-\frac{3}{5}\sin t\right)\hat{i} - \frac{4}{5}\hat{j} + \left(\frac{3}{5}\cos t\right)\hat{k}$

(2 points) 2. Find the following limit: $\lim\limits_{\substack{(x,y)\to(2,1)\\ x\neq 2}} \dfrac{xy-2y-2x+4}{x-2}$

$$= \lim_{\substack{(x,y)\to(2,1)\\ x\neq 2}} \frac{y(x-2)-2(x-2)}{x-2} = \lim_{\substack{(x,y)\to(2,1)\\ x\neq 2}} \frac{(x-2)(y-2)}{x-2} = \lim_{\substack{(x,y)\to(2,1)\\ x\neq 2}} y-2 = 1-2$$

$$= -1$$

(2 points) 3. Show that $f(x,y)=\dfrac{x^4}{x^4-y^2}$ has no limit as $(x,y)\to(0,0)$ by considering different paths of approach. Use $y=kx^2$, $k\neq 1$, and <u>explain</u> the result.

$$\lim_{\substack{(x,y)\to(0,0)\\ y=kx^2,\,k\neq 1}} \frac{x^4}{x^4-y^2} = \lim_{\substack{(x,y)\to(0,0)\\ y=kx^2,\,k\neq 1}} \frac{x^4}{x^4-(kx^2)^2} = \lim_{\substack{(x,y)\to(0,0)\\ y=kx^2,\,k\neq 1}} \frac{x^4}{x^4-k^2x^4} = \frac{1}{1-k^2}$$

Since the limit depends on k only, the limit does not exists

or if $k=2$, limit $= -\frac{1}{3}$ Since the limits are not equal given two
if $k=3$, limit $= -\frac{1}{8}$ different paths, the limit does not exist.

(3 points) 4. Given the following function: $f(x,y)=x^2+y^2$. Sketch the function's level curves when $k=0$, $k=4$, and $k=16$.

$k=0$ $x^2+y^2=0$

$k=4$ $x^2+y^2=4$

$k=16$ $x^2+y^2=16$