MATH 280 Quiz #2

#1) $\vec{r}(t) = \langle t^2, \sin t - t\cos t, \cos t + t\sin t \rangle, \ t > 0$

(4pts)

a.) $\vec{r}\,'(t) = \langle 2t, \cos t - \cos t + t\sin t, -\sin t + \sin t + t\cos t \rangle$

$= \langle 2t, t\sin t, t\cos t \rangle$

$|\vec{r}\,'(t)| = \sqrt{4t^2 + t^2\sin^2 t + t^2\cos^2 t} = t\sqrt{4 + \sin^2 t + \cos^2 t} = t\sqrt{5}$

$\hat{T} = \dfrac{\langle 2t, t\sin t, t\cos t \rangle}{t\sqrt{5}} = \langle \frac{2}{\sqrt{5}}, \frac{\sin t}{\sqrt{5}}, \frac{\cos t}{\sqrt{5}} \rangle$

$\hat{T}' = \langle 0, \frac{\cos t}{\sqrt{5}}, \frac{-\sin t}{\sqrt{5}} \rangle$

$|\hat{T}'| = \sqrt{0^2 + \frac{\cos^2 t}{5} + \frac{\sin^2 t}{5}} = \frac{1}{\sqrt{5}}$

$\hat{N} = \dfrac{\hat{T}'}{|\hat{T}'|} = \langle 0, \cos t, -\sin t \rangle$

(4pts)

b.) $\vec{B} = \hat{T} \times \hat{N} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{2}{\sqrt{5}} & \frac{\sin t}{\sqrt{5}} & \frac{\cos t}{\sqrt{5}} \\ 0 & \cos t & -\sin t \end{vmatrix} = \hat{\imath}\left(-\frac{\sin^2 t}{\sqrt{5}} - \frac{\cos^2 t}{\sqrt{5}}\right) - \hat{\jmath}\left(-\frac{2\sin t}{\sqrt{5}}\right) + \hat{k}\left(\frac{2\cos t}{\sqrt{5}}\right)$

$= -\frac{1}{\sqrt{5}}\hat{\imath} + \left(\frac{2\sin t}{\sqrt{5}}\right)\hat{\jmath} + \left(\frac{2\cos t}{\sqrt{5}}\right)\hat{k}$

(3pts)

#2) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^8 - y^8}{x^4 - y^4} = \lim\limits_{(x,y)\to(0,0)} \dfrac{(x^4+y^4)(x^4-y^4)}{x^4-y^4} = \lim\limits_{(x,y)\to(0,0)} x^4 + y^4 = 0$

(3pts)

#3) $f(x,y) = 9x^2 + y^2$

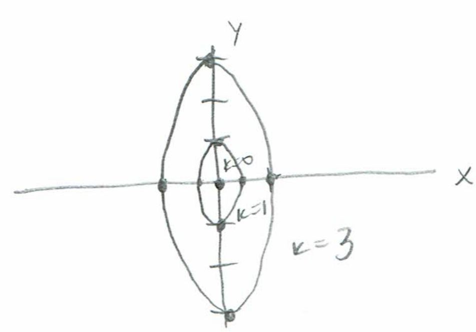
$k=0 \quad 9x^2 + y^2 = 0$

$k=1 \quad 9x^2 + y^2 = 1$

$\dfrac{x^2}{1/9} + y^2 = 1$

$k=9 \quad 9x^2 + y^2 = 9$

$x^2 + \dfrac{y^2}{9} = 1$



#5.) $f(x,y) = x^2y^3 \sin(x^4y^3)$

(2pts)
a.) $f_x = 2xy^3\sin(x^4y^3) + x^2y^3\cos(x^4y^3)(4x^3y^3)$

$= 2xy^3\sin(x^4y^3) + 4x^5y^6\cos(x^4y^3)$

(2pts)
b.) $f_y = 3x^2y^2\sin(x^4y^3) + x^2y^3\cos(x^4y^3)(3x^4y^2)$

$= 3x^2y^2\sin(x^4y^3) + 3x^6y^5\cos(x^4y^3)$

(3pts)
c.) $f_{xx} = 2y^3\sin(x^4y^3) + 2xy^3\cos(x^4y^3)\,4x^3y^3 + 20x^4y^6\cos(x^4y^3) + 4x^5y^6(-\sin(x^4y^3))(4x^3y^3)$

$= 2y^3\sin(x^4y^3) + 8x^4y^6\cos(x^4y^3) + 20x^4y^6\cos(x^4y^3) - 16x^8y^9\sin(x^4y^3)$

$= 2y^3\sin(x^4y^3) + 28x^4y^6\cos(x^4y^3) - 16x^8y^9\sin(x^4y^3)$

(3pts)
d.) $f_{xy} = 6xy^2\sin(x^4y^3) + 2xy^3\cos(x^4y^3)(3x^4y^2) + 24x^5y^5\cos(x^4y^3) + 4x^5y^6(-\sin(x^4y^3))(3x^4y^2)$

$= 6xy^2\sin(x^4y^3) + 6x^5y^5\cos(x^4y^3) + 24x^5y^5\cos(x^4y^3) - 12x^9y^8\sin(x^4y^3)$

$= 6xy^2\sin(x^4y^3) + 30x^5y^5\cos(x^4y^3) - 12x^9y^8\sin(x^4y^3)$

(3pts)
e.) $f_{yy} = 6x^2y\sin(x^4y^3) + 3x^2y^2\cos(x^4y^3)(3x^4y^2) + 15x^6y^4\cos(x^4y^3) + 3x^6y^5(-\sin(x^4y^3))(3x^4y^2)$

$= 6x^2y\sin(x^4y^3) + 9x^6y^4\cos(x^4y^3) + 15x^6y^4\cos(x^4y^3) - 9x^{10}y^7\sin(x^4y^3)$

$= 6x^2y\sin(x^4y^3) + 24x^6y^4\cos(x^4y^3) - 9x^{10}y^7\sin(x^4y^3)$

3pts)
#3 $f(x,y) = \dfrac{x^3}{x^3-y^6}$

x-axis
$y=0$ $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\y=0}} \frac{x^3}{x^3-y^6} = \lim_{\substack{(x,y)\to(0,0)\\y=0}} \frac{x^3}{x^3-0} = 1$

y-axis
$x=0$ $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\x=0}} \frac{x^3}{x^3-y^6} = \lim_{\substack{(x,y)\to(0,0)\\x=0}} \frac{0}{-y^6} = 0$

Since the limit approaches two different values along two different paths

The limit does not exist by the two paths approach