

Quiz #3

1.) $\cos(x^2y^3) + x^2 + y^4 - x^3y^5 = 0$

$f_x = -2xy^3 \sin(x^2y^3) + 2x - 3x^2y^5$

$f_y = -3x^2y^2 \sin(x^2y^3) + 4y^3 - 5x^3y^4$

4 $\frac{dy}{dx} = - \frac{-2xy^3 \sin(x^2y^3) + 2x - 3x^2y^5}{-3x^2y^2 \sin(x^2y^3) + 4y^3 - 5x^3y^4}$

2.) $f(x,y,z) = x^2y + y^2z$

$\vec{\nabla} f = (2xy)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2)\hat{k}$

$\vec{\nabla} f(1,2,3) = 4\hat{i} + 13\hat{j} + 4\hat{k}$

$\vec{u} = 2\hat{i} - \hat{j} + 2\hat{k}$

$|\vec{u}| = \sqrt{4+1+4} = 3$

$\hat{u} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

4 $\vec{\nabla} f \cdot \hat{u}$

$\langle 4, 13, 4 \rangle \cdot \langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \rangle$

$= \frac{8}{3} - \frac{13}{3} + \frac{8}{3} = 1$

3.) $f(x,y) = \sin(xy)$

$\vec{\nabla} f = y \cos(xy)\hat{i} + x \cos(xy)\hat{j}$

$\vec{\nabla} f(1,0) = \hat{j}$

2 a.) most rapid increase = \hat{j}

2 b.) most rapid decrease = $-\hat{j}$

2 c.) zero change = \hat{i} and $-\hat{i}$

4.) $x = y^2 + z^2 + 1$

$x - y^2 - z^2 - 1 = 0$

4 $\vec{\nabla} f = \hat{i} - 2y\hat{j} - 2z\hat{k}$

$\vec{\nabla} f(3,1,-1) = \hat{i} - 2\hat{j} + 2\hat{k}$

$1(x-3) - 2(y-1) + 2(z+1) = 0$

$x-3 - 2y+2 + 2z+2 = 0$

$x - 2y + 2z + 1 = 0$