1. $f(x,y) = x^2y^3\sin(x^4y^3)$

2pts a.) $f_x = 2xy^3\sin(x^4y^3) + x^2y^3(4x^3y^3)\cos(x^4y^3)$

$= 2xy^3\sin(x^4y^3) + 4x^5y^6\cos(x^4y^3)$

2pts b.) $f_y = 3x^2y^2\sin(x^4y^3) + x^2y^3(3x^4y^2)\cos(x^4y^3)$

$= 3x^2y^2\sin(x^4y^3) + 3x^6y^5\cos(x^4y^3)$

3pts c.) $f_{xx} = 2y^3\sin(x^4y^3) + 2xy^3(4x^3y^3)\cos(x^4y^3) + 20x^4y^6\cos(x^4y^3) + 4x^5y^6(4x^3y^3)(-\sin(x^4y^3))$

$= 2y^3\sin(x^4y^3) + 8x^4y^6\cos(x^4y^3) + 20x^4y^6\cos(x^4y^3) - 16x^8y^9\sin(x^4y^3)$

$= 2y^3\sin(x^4y^3) + 28x^4y^6\cos(x^4y^3) - 16x^8y^9\sin(x^4y^3)$

3pt d.) $f_{xy} = 6xy^2\sin(x^4y^3) + 2xy^3(3x^4y^2)\cos(x^4y^3) + 24x^5y^5\cos(x^4y^3) + 4x^5y^6(3x^4y^2)(-\sin(x^4y^3))$

$= 6xy^2\sin(x^4y^3) + 6x^5y^5\cos(x^4y^3) + 24x^5y^5\cos(x^4y^3) - 12x^9y^8\sin(x^4y^3)$

$= 6xy^2\sin(x^4y^3) + 30x^5y^5\cos(x^4y^3) - 12x^9y^8\sin(x^4y^3)$

3pts e.) $f_{yy} = 6x^2y\sin(x^4y^3) + 3x^2y^2(3x^4y^2)\cos(x^4y^3) + 15x^6y^4\cos(x^4y^3) + 3x^6y^5(3x^4y^2)(-\sin(x^4y^3))$

$= 6x^2y\sin(x^4y^3) + 9x^6y^4\cos(x^4y^3) + 15x^6y^4\cos(x^4y^3) - 9x^{10}y^7\sin(x^4y^3)$

$= 6x^2y\sin(x^4y^3) + 24x^6y^4\cos(x^4y^3) - 9x^{10}y^7\sin(x^4y^3)$

3pts 2.) $w = x^2y^5 + z^3$, $x = 6t^2+4t+5$, $y = 2t^3+7t-8$, $z = 5t^3+8t-4$

$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$

$= (2xy^5)(12t+4) + (5x^2y^4)(6t^2+7) + 3z^2(15t^2+8)$

4pts 3.) $f(x,y) = x^2 - xy - y^2$ $f_x = 2x - y$ $f_y = -x - 2y$

$z+1 = 1(x-1) - 3(y-1)$ $z+1 = x-1-3y+3$ $z+1 = x-3y+2$ $z = x-3y+1$

$x = 1+t$ $z = -1+t$
$y = 1-3t$

5pts 4.) $f(x,y) = \frac{1+y}{1+x}$ $f_x = -\frac{1+y}{(1+x)^2}$ $f_y = \frac{1}{1+x}$ $f_x(1,3) = \frac{4}{(2)^2} = 1$ $f_y(1,3) = \frac{1}{2}$ $z = \frac{4}{2} = 2$

$z-2 = 1(x-1) + \frac{1}{2}(y-3)$ $z-2 = x-1+\frac{1}{2}y-\frac{3}{2}$ $z-2 = x+\frac{1}{2}y-\frac{1}{2}$ $z = x+\frac{1}{2}y+\frac{3}{2}$

$z = 1.2 + \frac{1}{2}(3.1) + \frac{3}{2}$

$L(1.2, 3.1) = 1.85$