MATH 280 – QUIZ #3 Name: Key

Directions: This is a take-home quiz. It is due at the beginning of class on Wednesday, April 25, 2018. Please show all work for maximum credit. This quiz is worth 14 points. Good luck!

(3 points) 1. Find the derivative of the function $f(x,y) = 2x^2 + y^2$ at $P_0(-1,2)$ in the direction of $\vec{A} = 3\hat{i} - 5\hat{j}$.

$\vec{A} = 3\hat{i} - 5\hat{j}$

$|\vec{A}| = \sqrt{3^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34}$

$\hat{u} = \frac{\vec{A}}{|\vec{A}|} = \frac{3}{\sqrt{34}}\hat{i} - \frac{5}{\sqrt{34}}\hat{j}$

$\vec{\nabla} f = 4x\hat{i} + 2y\hat{j}$

$\vec{\nabla} f(-1,2) = -4\hat{i} + 4\hat{j}$

$D_{\hat{u}} f(-1,2) = \vec{\nabla} f(-1,2) \cdot \hat{u}$

$= (-4\hat{i} + 4\hat{j}) \cdot \left(\frac{3}{\sqrt{34}}\hat{i} - \frac{5}{\sqrt{34}}\hat{j}\right)$

$= -\frac{12}{\sqrt{34}} - \frac{20}{\sqrt{34}} = \frac{-32}{\sqrt{34}}$

(3 points) 2. Find the equation of the tangent plane and the equation of the normal line at the point $P_0(1,-1,3)$ on the surface $x^2 + 2xy - y^2 + z^2 = 7$.

$F(x,y,z) = x^2 + 2xy - y^2 + z^2 = 7$

$\vec{\nabla} F = (2x + 2y)\hat{i} + (2x - 2y)\hat{j} + (2z)\hat{k}$

$\vec{\nabla} F(1,-1,3) = 0\hat{i} + 4\hat{j} + 6\hat{k}$

Tangent Plane:

$0(x-1) + 4(y+1) + 6(z-3) = 0$

$4y + 4 + 6z - 18 = 0$

$4y + 6z - 14 = 0$

Normal Line

$x = 1$

$y = -1 + 4t$

$z = 3 + 6t$

(4 points) 3. Find all the local maxima, local minima, and saddle points of the given function.

$$f(x) = 2xy - x^2 - 2y^2 + 3x + 4$$

$f_x = 2y - 2x + 3$ $f_y = 2x - 4y$

$2y - 2x + 3 = 0$ AND $2x - 4y = 0$

$-2x + 2y = -3$
$2x - 4y = 0$
$-2y = -3$
$y = 3/2$

$2x - 4(\frac{3}{2}) = 0$
$2x = 6$
$x = 3$

$(3, 3/2)$

$f_{xx} = -2$ $f_{yy} = -4$ $f_{xy} = 2$

$D = (-2)(-4) - (2)^2 = 4 > 0$ $f_{xx} = -2 < 0$ local maximum.

(4 points) 4. Find the minimum and maximum values of $f(x,y) = 3x - y + 6$ subject to the constraint $x^2 + y^2 = 4$.

$f(x,y) = 3x - y + 6$ $g(x,y) = x^2 + y^2 = 4$

$\vec{\nabla} f = 3\hat{i} - \hat{j}$ $\vec{\nabla} g = 2x\hat{i} + 2y\hat{j}$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$3 = (2x)\lambda$ $-1 = (2y)\lambda$ $x^2 + y^2 = 4$

$\frac{3}{2\lambda} = x$ $\frac{-1}{2\lambda} = y$ $\left(\frac{3}{2\lambda}\right)^2 + \left(\frac{-1}{2\lambda}\right)^2 = 4$

$\frac{9}{4\lambda^2} + \frac{1}{4\lambda^2} = 4$

$9 + 1 = 4(4\lambda^2)$

$10 = 16\lambda^2$

$\lambda^2 = \frac{10}{16}$

$\lambda = \pm\frac{\sqrt{10}}{4}$

$x = \frac{3}{2(\pm\frac{\sqrt{10}}{4})} = \pm\frac{6}{\sqrt{10}}$

$y = \frac{-1}{2(\pm\frac{\sqrt{10}}{4})} = \frac{2}{\mp\sqrt{10}}$

$f\left(\frac{6}{\sqrt{10}}, \frac{2}{\sqrt{10}}\right) = \frac{18}{\sqrt{10}} - \frac{2}{\sqrt{10}} + 6 = \frac{16}{\sqrt{10}} + 6$

$f\left(\frac{6}{\sqrt{10}}, -\frac{2}{\sqrt{10}}\right) = \frac{18}{\sqrt{10}} + \frac{2}{\sqrt{10}} + 6 = \frac{20}{\sqrt{10}} + 6$ max

$f\left(-\frac{6}{\sqrt{10}}, \frac{2}{\sqrt{10}}\right) = -\frac{18}{\sqrt{10}} - \frac{2}{\sqrt{10}} + 6 = -\frac{20}{\sqrt{10}} + 6$ min

$f\left(-\frac{6}{\sqrt{10}}, -\frac{2}{\sqrt{10}}\right) = -\frac{18}{\sqrt{10}} + \frac{2}{\sqrt{10}} + 6 = -\frac{16}{\sqrt{10}} + 6$