MATH 280 – QUIZ #3 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 14 points. Good luck!

1. Find $f_{xx}, f_{xy}, f_{yx}, f_{yy}$: Given $f(x,y) = 4x^3\cos(3y^2)$. Determine the following.

(1 point) a. $f_x = 12x^2\cos(3y^2)$

(1 point) b. $f_y = -4x^3(6y)\sin(3y^2) = -24x^3 y\sin(3y^2)$

(1 point) c. $f_{xx} = 24x\cos(3y^2)$

(1 point) d. $f_{xy} = -12x^2(6y)\sin(3y^2) = -72x^2 y\sin(3y^2)$

(1 point) e. $f_{yy} = -24x^3\sin(3y^2) - 24x^3 y(6y)\cos(3y^2)$

$= -24x^3\sin(3y^2) - 144x^3y^2\cos(3y^2)$

(3 points) 2. Find the following limit: $\lim\limits_{\substack{(x,y)\to(3,6)\\ x+y\neq 9}} \dfrac{x+y-9}{\sqrt{x+y}-3} \cdot \dfrac{\sqrt{x+y}+3}{\sqrt{x+y}+3}$

$$= \lim_{\substack{(x,y)\to(3,6)\\ x+y\neq 9}} \frac{(x+y-9)(\sqrt{x+y}+3)}{x+y-9}$$

$$= \lim_{\substack{(x,y)\to(3,6)\\ x+y\neq 9}} (\sqrt{x+y}+3) = \sqrt{3+6}+3 = 3+3 = 6$$

(3 points) 3. Given the following function: $f(x,y)=x^2+y^2$. Sketch the function's level curves when $k=0$, $k=9$, and $k=25$.

$k=0$

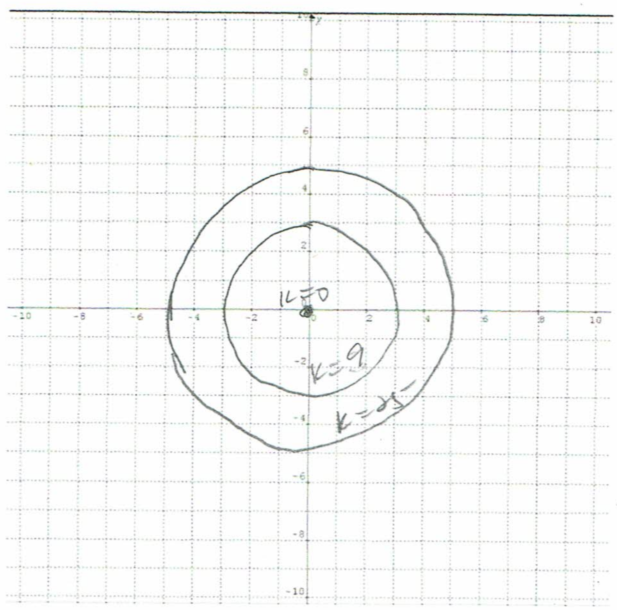
$x^2+y^2=0$

$k=9$

$x^2+y^2=9$

$k=25$

$x^2+y^2=25$



(3 points) 4. Show that $\lim_{(x,y)\to(0,0)} \frac{x^6}{x^6-y^2}$ has no limit by considering the different paths method. Explain your answer.

$y = kx^3,\ k \neq 1$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=kx^3}} \frac{x^6}{x^6-(kx^3)^2} = \lim_{\substack{(x,y)\to(0,0)\\ y=kx^3}} \frac{x^6}{x^6-k^2x^6} = \frac{1}{1-k^2},\ k \neq 1$$

Since the limit depends on k, which is based on the path taken to (0,0), the limit is path dependent. Therefore, the limit does not exist.

OR $k=2$ limit $= \frac{1}{1-2^2} = -\frac{1}{3}$

$k=3$ limit $= \frac{1}{1-3^2} = -\frac{1}{8}$

Since there are two different values for the limit based on two different paths, the limit does not exist.