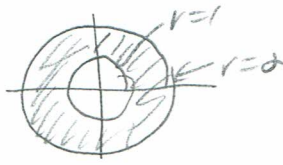


Quiz #4

1.) $z = \sqrt{x^2 + y^2}$ $1 \leq x^2 + y^2 \leq 4$

$z = \sqrt{r^2} = r$



4
$$\int_0^{2\pi} \int_1^2 r \, r \, dr \, d\theta = \int_0^{2\pi} \int_1^2 r^2 \, dr \, d\theta = \int_0^{2\pi} \left. \frac{r^3}{3} \right|_1^2 \, d\theta = \int_0^{2\pi} \frac{7}{3} \, d\theta = \frac{7}{3} \theta \Big|_0^{2\pi} = \frac{14\pi}{3}$$

2. $y = 1 - x^2$ $y = 0$ $\rho(x, y) = ky$

4
$$M_x = \int_{-1}^1 \int_0^{1-x^2} y \, ky \, dy \, dx = k \int_{-1}^1 \int_0^{1-x^2} y^2 \, dy \, dx = k \int_{-1}^1 \left. \frac{y^3}{3} \right|_0^{1-x^2} \, dx = \frac{k}{3} \int_{-1}^1 (1-x^2)^3 \, dx$$

$$= \frac{k}{3} \int_{-1}^1 (1 - 3x^2 + 3x^4 - x^6) \, dx = \frac{k}{3} \left(x - x^3 + \frac{3}{5}x^5 - \frac{x^7}{7} \right) \Big|_{-1}^1$$

$$= \frac{k}{3} \left((1 - 1 + \frac{3}{5} - \frac{1}{7}) - (-1 + 1 - \frac{3}{5} + \frac{1}{7}) \right) = \frac{k}{3} \left(\frac{6}{5} - \frac{2}{7} \right) = \frac{k}{3} \left(\frac{32}{35} \right) = \frac{32}{105} k$$

4
$$M_y = \int_{-1}^1 \int_0^{1-x^2} x \, ky \, dy \, dx = k \int_{-1}^1 x \left. \frac{y^2}{2} \right|_0^{1-x^2} \, dx = \frac{k}{2} \int_{-1}^1 x(1-x^2)^2 \, dx = \frac{k}{2} \left(\frac{-1}{2} \right) \int_{-1}^1 u^2 \, du$$

$$= -\frac{1}{4} k \left. \frac{u^3}{3} \right|_{-1}^1 = -\frac{1}{12} k (1-x^2) \Big|_{-1}^1 = 0$$

4
$$M = \int_{-1}^1 \int_0^{1-x^2} ky \, dy \, dx = k \int_{-1}^1 \left. \frac{y^2}{2} \right|_0^{1-x^2} \, dx = \frac{k}{2} \int_{-1}^1 (1-x^2)^2 \, dx = \frac{k}{2} \int_{-1}^1 (1 - 2x^2 + x^4) \, dx$$

$$= \frac{k}{2} \left(x - \frac{2}{3}x^3 + \frac{x^5}{5} \right) \Big|_{-1}^1 = \frac{k}{2} \left((1 - \frac{2}{3} + \frac{1}{5}) - (-1 + \frac{2}{3} - \frac{1}{5}) \right) = \frac{k}{2} \left(2 - \frac{4}{3} + \frac{2}{5} \right) = \frac{k}{2} \left(\frac{30 - 20 + 6}{15} \right)$$

$$= \frac{k}{2} \left(\frac{16}{15} \right) = \frac{8}{15} k$$

2
$$\bar{x} = \frac{M_y}{M} = \frac{0}{\frac{8}{15}k} = 0$$

$$\bar{y} = \frac{M_x}{M} = \frac{\frac{32}{105}k}{\frac{8}{15}k} = \frac{\frac{32}{105}}{\frac{8}{15}} = \frac{32}{105} \cdot \frac{15}{8} = \frac{4}{7} \quad (0, \frac{4}{7})$$

3.)

$$\int_D \int \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{1}{2}\right)^2 + 1} dA$$

$$= \int_0^{2\pi} \int_0^4 \sqrt{\frac{9}{16} + \frac{1}{4} + 1} r dr d\theta$$

$$4 = \frac{\sqrt{29}}{4} \int_0^{2\pi} \frac{r^2}{2} \Big|_0^4 d\theta = 2\sqrt{29} \int_0^{2\pi} d\theta$$

$$= 2\sqrt{29} \theta \Big|_0^{2\pi} = 4\pi\sqrt{29}$$

