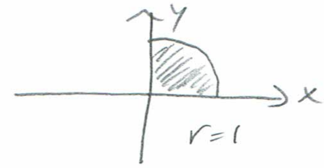
MATH 280 – QUIZ #4 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 14 points. Good luck!

(4 points) 1. Change the following Cartesian integral into an equivalent polar integral. Then evaluate the polar integral.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-\sqrt{x^2+y^2}}\, dy\, dx$$



$y=\sqrt{1-x^2}$

$x^2+y^2=1$

$$\int_0^{\pi/2}\int_0^1 e^{-r} r\, dr\, d\theta = \int_0^{\pi/2} (-re^{-r}-e^{-r})\Big|_0^1 = \int_0^{\pi/2} [(-e^{-1}-e^{-1})-(-0-e^0)]\, d\theta$$

$u=r \quad du=e^{-r}dr$

$du=dr \quad u=-e^{-r}$

$\int re^{-r}dr = -re^{-r}+\int e^{-r}dr$

$= -re^{-r}-e^{-r}+C$

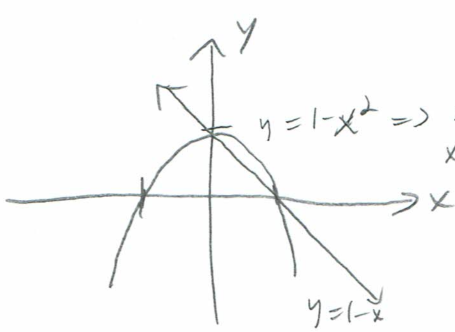
$$= \int_0^{\pi/2} (-2e^{-1}+1)\, d\theta$$

$$= \left(-\frac{2}{e}+1\right)\theta \Big|_0^{\pi/2} = \left(-\frac{2}{e}+1\right)\frac{\pi}{2}$$

$$= -\frac{\pi}{e}+\frac{\pi}{2}$$

(2 points) 2. For the following integral, sketch the region over which the double integral is evaluated. Then, write an equivalent double integral with the order of integration reversed.

$$\int_0^1 \int_{1-x}^{1-x^2} dy\, dx$$



$y=1-x^2 \Rightarrow x^2=1-y$

$x=\pm\sqrt{1-y}$

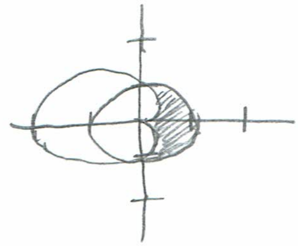
$\Rightarrow x=1-y$

$$\int_0^1 \int_{1-y}^{\sqrt{1-y}} dx\, dy$$

(4 points) 3. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$.

$r = 1 - \cos\theta$

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	0	1	2	1	0



$1 - \cos\theta = 1$

$0 = \cos\theta$

$\theta = \pi/2, -\pi/2$

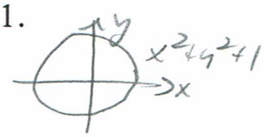
$$\int_{-\pi/2}^{\pi/2}\int_{1-\cos\theta}^{1} r\,dr\,d\theta = \int_{-\pi/2}^{\pi/2} \frac{r^2}{2}\Big|_{1-\cos\theta}^{1} d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2}\left[(1)^2 - (1-\cos\theta)^2\right] d\theta = \frac{1}{2}\int_{-\pi/2}^{\pi/2}\left[1 - (1 - 2\cos\theta + \cos^2\theta)\right] d\theta = \frac{1}{2}\int_{-\pi/2}^{\pi/2}(2\cos\theta - \cos^2\theta)\,d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2}\left(2\cos\theta - \frac{1+\cos 2\theta}{2}\right) d\theta = \frac{1}{2}\left[2\sin\theta - \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_{-\pi/2}^{\pi/2} = \frac{1}{2}\left[\left(2 - \frac{\pi}{4}\right) - \left(-2 + \frac{\pi}{4}\right)\right]$$

$$= \frac{1}{2}\left(4 - \frac{\pi}{2}\right) = 2 - \frac{\pi}{4}$$

(4 points) 4. Find the volume of the region bounded below by the paraboloid $z = x^2 + y^2$, laterally by the cylinder $x^2 + y^2 = 1$, and above by the paraboloid $z = x^2 + y^2 + 1$.



$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{x^2+y^2+1} dz\,dy\,dx$$ convert to polar

$$\int_0^{2\pi}\int_0^1\int_{r^2}^{r^2+1} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 rz\Big|_{r^2}^{r^2+1} dr\,d\theta = \int_0^{2\pi}\int_0^1 r(r^2+1-r^2)\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 r\,dr\,d\theta = \int_0^{2\pi}\frac{r^2}{2}\Big|_0^1 d\theta = \frac{1}{2}\int_0^{2\pi} d\theta = \frac{1}{2}\theta\Big|_0^{2\pi} = \pi$$