MATH 280 – QUIZ #4 *Name:* Key

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

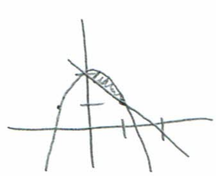
(5 points) *1.* Evaluate the following integral: $\int_0^{\pi}\int_0^{\sin x} y\,dy\,dx$

$$\int_0^{\pi} \frac{y^2}{2}\Big|_0^{\sin x} dx = \frac{1}{2}\int_0^{\pi} \sin^2 x\,dx = \frac{1}{2}\int_0^{\pi}\left(\frac{1-\cos 2x}{2}\right) dx = \frac{1}{4}\int_0^{\pi}(1-\cos 2x)\,dx$$

$$= \frac{1}{4}\left(x - \frac{\sin 2x}{2}\right)\Big|_0^{\pi} = \frac{\pi}{4}$$

(5 points) *2.* Find the volume of the region under the surface $z = x^2$ and over the region D enclosed by the parabola $y = 2 - x^2$ and the line $x + y = 2$ in the *xy*-plane.

$$2 - x^2 = 2 - x$$
$$0 = x^2 - x$$
$$0 = x(x-1)$$
$$x = 0, 1$$



$$\int_0^1 \int_{2-x}^{2-x^2} x^2\,dy\,dx = \int_0^1 x^2 y \Big|_{2-x}^{2-x^2} dx$$

$$= \int_0^1 x^2\left[(2-x^2)-(2-x)\right] dx = \int_0^1 \left[x^2(-x^2+x)\right] dx$$

$$= \int_0^1 (-x^4 + x^3)\,dx = \left(-\frac{x^5}{5} + \frac{x^4}{4}\right)\Big|_0^1$$

$$= -\frac{1}{5} + \frac{1}{4} = \frac{-4+5}{20} = \frac{1}{20}$$

(6 points) *3.* Use Lagrange multipliers to find the minimum and maximum values of $f(x,y) = xy$ subject to the constraint $x^2 + y^2 = 10$.

$\vec{\nabla} f = y\hat{\imath} + x\hat{\jmath}$ $\vec{\nabla} g = 2x\hat{\imath} + 2y\hat{\jmath}$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$y = 2x\lambda$ $x = 2y\lambda$ $x^2 + y^2 = 10$

$\frac{y}{2x} = \lambda$ $x = 2y\left(\frac{y}{2x}\right)$

$2x^2 = 2y^2$

$x^2 = y^2$ $x^2 + x^2 = 10$

$2x^2 = 10$

$x^2 = 5$

$x = \pm\sqrt{5}$

$x = \sqrt{5}$ $y^2 = 5 \Rightarrow y = \pm\sqrt{5}$

$(\sqrt{5}, \sqrt{5})$ $(\sqrt{5}, -\sqrt{5})$

$x = -\sqrt{5}$ $y^2 = 5 \Rightarrow y = \pm\sqrt{5}$

$(-\sqrt{5}, \sqrt{5})$ $(-\sqrt{5}, -\sqrt{5})$

$f(\sqrt{5}, \sqrt{5}) = 5$

$f(\sqrt{5}, -\sqrt{5}) = -5$

$f(-\sqrt{5}, \sqrt{5}) = -5$

$f(-\sqrt{5}, -\sqrt{5}) = 5$

minimum value $= -5$

maximum value $= 5$.