MATH 280 – QUIZ #4 *Name:* KEY

Directions: This is a take home quiz. This quiz is due at the beginning of class on Thursday, November 7, 2019. Please show all work for maximum credit. This quiz is worth 14 points. Good luck!

(4 points) 1. Find all the local maxima, local minima, and saddle points of the given function.

$$f(x,y) = 2xy - x^2 - 2y^2 + 3x + 4$$

$f_x = 2y - 2x + 3$ $f_y = 2x - 4y$

$2y - 2x + 3 = 0$ AND $2x - 4y = 0$

$x = 2y$

$2y - 4y + 3 = 0$

$-2y + 3 = 0$

$3 = 2y$

$y = 3/2$ $x = 3$ $(3, 3/2)$

$f_{xx} = -2$ $f_{xy} = 2$ $f_{yy} = -4$

$D = f_{xx}f_{yy} - (f_{xy})^2 = (-2)(-4) - (2)^2 = 4 > 0$

$f_{xx} = -2 < 0$ local max at $(3, 3/2)$

(3 points) 2. Evaluate the following integral on the given rectangle.

$$\iint_R xye^x \, dydx \qquad R: \ 0 \le x \le 1, \ 1 \le y \le 2$$

$\int_0^1 \int_1^2 xye^x \, dy\,dx = \int_0^1 xe^x \frac{y^2}{2}\Big|_1^2 dx = \int_0^1 xe^x (2 - \tfrac{1}{2}) dx = \frac{3}{2}\int_0^1 xe^x dx$

$= \frac{3}{2}(xe^x - e^x)\Big|_0^1 = \frac{3}{2}((1e - e) - (0 - 1)) = 3/2$

(4 points) *3.* Find the minimum and maximum values of $f(x,y) = 3x - y + 6$ subject to the constraint $x^2 + y^2 = 4$.

$\vec{\nabla} f = 3\hat{i} - \hat{j}$ $\quad \vec{\nabla} g = 2x + 2y$

$\vec{\nabla} f = \lambda \vec{\nabla} g$ and $x^2 + y^2 = 4$

$3 = \lambda(2x)$ $\quad -1 = \lambda(2y)$ $\quad x^2 + y^2 = 4$

$x = \frac{3}{2\lambda}$ $\quad y = -\frac{1}{2\lambda}$ $\quad \left(\frac{3}{2\lambda}\right)^2 + \left(-\frac{1}{2\lambda}\right)^2 = 4$

$\frac{9}{4\lambda^2} + \frac{1}{4\lambda^2} = 4$

$10 = 16\lambda^2$

$\lambda^2 = \frac{10}{16}$

$\lambda = \pm \frac{\sqrt{10}}{4}$

$\lambda = \frac{\sqrt{10}}{4}$ $\quad x = \frac{3}{2\left(\frac{\sqrt{10}}{4}\right)}$ $\quad y = -\frac{1}{2\left(\frac{\sqrt{10}}{4}\right)}$

$x = \frac{6}{\sqrt{10}}$ $\quad y = -\frac{2}{\sqrt{10}}$

$\lambda = -\frac{\sqrt{10}}{4}$ $\quad x = -\frac{6}{\sqrt{10}}$ $\quad y = \frac{2}{\sqrt{10}}$

$f\left(\frac{6}{\sqrt{10}}, -\frac{2}{\sqrt{10}}\right) = 3\left(\frac{6}{\sqrt{10}}\right) - \left(-\frac{2}{\sqrt{10}}\right) + 6$

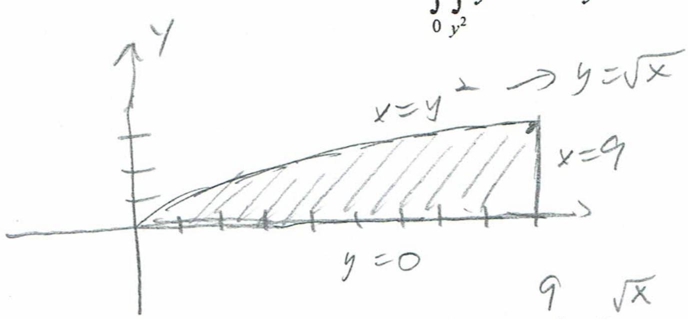
$= \frac{20}{\sqrt{10}} + 6$ max

$f\left(-\frac{6}{\sqrt{10}}, \frac{2}{\sqrt{10}}\right) = 3\left(-\frac{6}{\sqrt{10}}\right) - \frac{2}{\sqrt{10}} + 6$

$= -\frac{20}{\sqrt{10}} + 6$ min

(3 points) *4.* Sketch the region of integration and reverse the order of integration.

$$\int_0^3 \int_{y^2}^9 y e^{-x^2}\, dx\, dy$$



$$\int_0^9 \int_0^{\sqrt{x}} y e^{-x^2}\, dy\, dx$$