MATH 280 – QUIZ #~~3~~4 Name: KEY

Directions: Please show all work for maximum credit. This quiz is worth 14 points. Good luck!

(3 points) 1. Given $w = x^3yz^2 + y^2z$, $x = 4t-1$, $y = 3t^2+2$, $z = 2t+1$. Find $\frac{dw}{dt}$.

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$= (3x^2yz^2)(4) + (x^3z^2 + 2yz)(6t) + (2x^3yz + y^2)(2)$$

$$= 12(4t-1)^2(3t^2+2)(2t+1) + 6t\left((4t-1)^3(2t+1)^2 + 2(3t^2+2)(2t+1)\right)$$
$$+ 2\left(2(4t-1)^3(3t^2+2)(2t+1) + (3t^2+2)\right)$$

(3 points) 2. Find the derivative of the function $f(x,y) = \frac{x-y}{xy+2}$ at $P_0(1,-1)$ in the direction of $\vec{u} = 12\hat{i} + 5\hat{j}$.

$$|\vec{u}| = \sqrt{144+25} = \sqrt{169} = 13$$

$$\hat{u} = \frac{12}{13}\hat{i} + \frac{5}{13}\hat{j}$$

$$\vec{\nabla} f = \frac{(1)(xy+2) - y(x-y)}{(xy+2)^2}\hat{i} + \frac{(-1)(xy+2) - x(x-y)}{(xy+2)^2}\hat{j}$$

$$\vec{\nabla} f\Big|_{(1,-1)} = \frac{(-1+2)+1(2)}{(-1+2)^2}\hat{i} + \frac{(-1)(-1+2) - 1(2)}{(-1+2)^2}\hat{j}$$

$$= 3\hat{i} - 3\hat{j}$$

$$\vec{\nabla} f \cdot \hat{u} = (3\hat{i} - 3\hat{j})\cdot\left(\frac{12}{13} + \frac{5}{13}\right) = \frac{36}{13} - \frac{15}{13} = \frac{21}{13}$$

(4 points) 3. Find the equation of the tangent plane and the equation of the normal line at the point $P_0(3,5,-4)$ on the surface $x^2+y^2-z^2=18$.

$f_x = 2x \quad f_y = 2y \quad f_z = -2z$

$f_x|_{P_0} = 6 \quad f_y|_{P_0} = 10 \quad f_z|_{P_0} = 8$

Tangent Plane:

$6(x-3)+10(y-5)+8(z+4)=0$

$6x-18+10y-50+8z+32=0$

$6x+10y+8z-36=0$

$3x+5y+4z=18$

Normal Line

$x=3+6t \qquad x=3+3t$

$y=5+10t \quad \text{OR} \quad y=5+5t$

$z=-4+8t \qquad z=-4+4t$

(4 points) 4. Find all the local maxima, local minima, and saddle points of the given function.

$$f(x)=2x^2+3xy+4y^2-5x+2y$$

$f_x = 4x+3y-5 \qquad f_y = 3x+8y+2$

$4x+3y-5=0$ AND $3x+8y+2=0$

$3(4x+3y=5) \qquad 12x+9y=15$

$-4(3x+8y=-2) \qquad -12x-32y=8$

$-23y=23$

$y=-1$

$4x+3(-1)=5$

$4x-3=5$

$4x=8$

$x=2$

Critical point $(2,-1)$

$f_{xx}=4 \quad f_{yy}=8 \quad f_{xy}=3$

$D=(4)(8)-(3)^2=32-9=23>0$

$f_{xx}=4>0$

$(2,-1)$ local minimum.