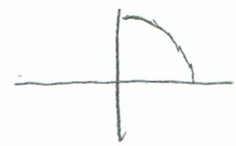
MATH 280 – QUIZ #5 *Name:* Key

Directions: Please show all work for maximum credit. This quiz is worth 14 points. Good luck!

(4 points) 1. Change the following Cartesian integral into an equivalent polar integral. Then evaluate the polar integral.

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \left(x^2+y^2\right) dxdy$$



$x = \sqrt{1-y^2}$

$x^2+y^2=1$

$r=1$

$$\int_0^{\pi/2}\int_0^1 r^2 r\, dr\, d\theta = \int_0^{\pi/2}\int_0^1 r^3\, dr\, d\theta$$

$$= \int_0^{\pi/2} \frac{r^4}{4}\Big|_0^1 d\theta = \int_0^{\pi/2} \frac{1}{4}\, d\theta = \frac{1}{4}\theta\Big|_0^{\pi/2} = \pi/8$$

(3 points) 2. Evaluate the following integral.

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz\ dy\ dx$$

$$= \int_0^1 \int_0^{1-x} z\Big|_0^{1-x-y} dy\,dx = \int_0^1 \int_0^{1-x} (1-x-y)\, dy\,dx$$

$$= \int_0^1 \left(y - xy - \frac{y^2}{2}\right)\Big|_0^{1-x} dx = \int_0^1 \left((1-x) - x(1-x) - \frac{(1-x)^2}{2}\right) dx$$

$$= \int_0^1 \left(1-x-x+x^2 - \frac{1-2x+x^2}{2}\right) dx = \int_0^1 \left(\frac{1}{2} - x + \frac{1}{2}x^2\right) dx$$

$$= \left(\frac{1}{2}x - \frac{1}{2}x^2 + \frac{1}{6}x^3\right)\Big|_0^1 = \frac{1}{6}$$

(4 points) 3. Find the volume of the region in the first octant bounded by the coordinate planes, the plane $y+z=3$, and the cylinder $x^2+y^2=9$.

$$\int_0^{\pi/2}\int_0^{3}\int_0^{3-y} r\,dz\,dr\,d\theta$$

$$= \int_0^{\pi/2}\int_0^{3}\int_0^{3-r\sin\theta} r\,dz\,dr\,d\theta = \int_0^{\pi/2}\int_0^{3} rz\Big|_0^{3-r\sin\theta} dr\,d\theta = \int_0^{\pi/2}\int_0^{3} r(3-r\sin\theta)\,dr\,d\theta$$

$$= \int_0^{\pi/2}\int_0^{3}(3r - r^2\sin\theta)\,dr\,d\theta = \int_0^{\pi/2}\left(\frac{3}{2}r^2 - \frac{r^3}{3}\sin\theta\right)\Big|_0^{3} d\theta$$

$$= \int_0^{\pi/2}\left(\frac{27}{2} - \frac{27}{3}\sin\theta\right) d\theta = \left(\frac{27}{2}\theta + \frac{27}{3}\cos\theta\right)\Big|_0^{\pi/2}$$

$$= \frac{27\pi}{4} - 9$$

(3 points) 4. Find the volume of the region cut from the solid sphere $\rho \le 3$ by the half-planes $\theta = 0$ and $\theta = \pi/6$ in the first octant.

$$\int_0^{\pi/6}\int_0^{\pi/2}\int_0^{3} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \int_0^{\pi/6}\int_0^{\pi/2}\frac{\rho^3}{3}\sin\phi\Big|_0^{3} d\phi\,d\theta$$

$$= 9\int_0^{\pi/6}\int_0^{\pi/2}\sin\phi\,d\phi\,d\theta = 9\int_0^{\pi/6}(-\cos\phi)\Big|_0^{\pi/2} d\theta$$

$$= 9\int_0^{\pi/6} d\theta = 9\theta\Big|_0^{\pi/6} = 3\pi/2$$