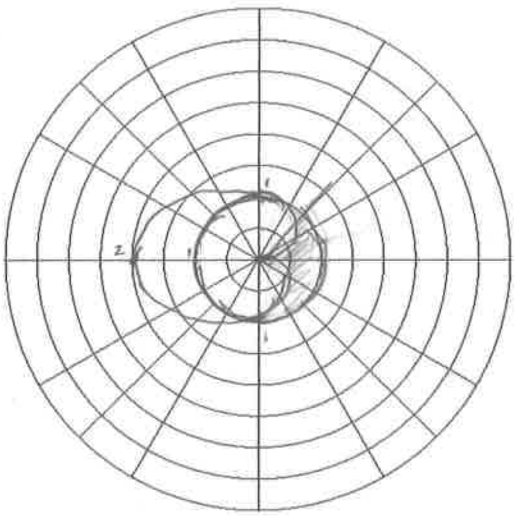
MATH 280 – QUIZ #5 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz will be taken out of 14 points. Good luck!

(5 points) 1. Find the area of the region that lies inside the circle $r=1$ and outside the cardioid $r=1-\cos\theta$.



θ	0	$\pi/2$	π	$3\pi/2$	2π
r	0	1	2	1	0

$$\int_{-\pi/2}^{\pi/2}\int_{1-\cos\theta}^{1} r\,dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \frac{r^2}{2}\Big|_{1-\cos\theta}^{1} d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \left[(1)^2 - (1-\cos\theta)^2\right] d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \left[1-(1-2\cos\theta+\cos^2\theta)\right] d\theta = \frac{1}{2}\int_{-\pi/2}^{\pi/2} (2\cos\theta - \cos^2\theta)\, d\theta = \frac{1}{2}\int_{-\pi/2}^{\pi/2}\left(2\cos\theta - \frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2}\left(2\cos\theta - \frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta = \frac{1}{2}\left(2\sin\theta - \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2}\left[\left(2-\frac{\pi}{4}\right) - \left(-2+\frac{\pi}{4}\right)\right] = \frac{1}{2}\left[4 - \frac{\pi}{2}\right] = 2 - \frac{\pi}{4}$$

(5 points) 2. Find the surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the *xy*-plane.

D: $4 - x^2 - y^2 = 0$

$x^2 + y^2 = 4 \Leftarrow r = 2$

$$\iint_D \sqrt{(-2x)^2 + (-2y)^2 + 1}\, dA$$

$$= \iint_D \sqrt{4x^2 + 4y^2 + 1}\, dA = \int_0^{2\pi}\int_0^2 \sqrt{4r^2+1}\, r\, dr\, d\theta$$

$u = 4r^2 + 1$

$du = 8r\, dr$

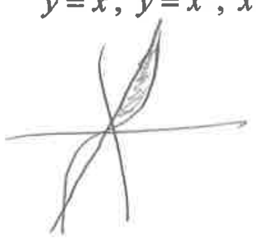
$\frac{1}{8} du = r\, dr$

$$= \frac{1}{8}\int_0^{2\pi}\int_0^2 u^{1/2}\, du\, d\theta = \frac{1}{8}\cdot\frac{2}{3}\int_0^{2\pi} u^{3/2}\Big|_0^2\, d\theta$$

$$= \frac{1}{8}\int_0^{2\pi} (4r^2+1)^{3/2}\Big|_0^2\, d\theta = \frac{1}{8}\int_0^{2\pi} (17^{3/2} - 1)\, d\theta = \frac{1}{8}(17^{3/2} - 1)\,\theta\Big|_0^{2\pi}$$

$$= \frac{1}{8}(17^{3/2} - 1)(2\pi) = \frac{\pi}{4}(17^{3/2} - 1)$$

(5 points) 3. Evaluate the double integral $\iint_D (x^2 + 2y)\, dA$ where D is the region bounded by $y = x$, $y = x^3$, $x \geq 0$.



$$\int_0^1\int_{x^3}^{x} (x^2 + 2y)\, dy\, dx = \int_0^1 (x^2 y + y^2)\Big|_{x^3}^{x}\, dx$$

$x^3 = x$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x+1)(x-1) = 0$

$x = 0, -1, 1$

$$= \int_0^1 [(x^3 + x^2) - (x^5 + x^6)]\, dx = \int_0^1 (x^3 + x^2 - x^5 - x^6)\, dx$$

$$= \left(\frac{x^4}{4} + \frac{x^3}{3} - \frac{x^6}{6} - \frac{x^7}{7}\right)\Big|_0^1 = \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{7}$$

$$= \frac{21 + 28 - 14 - 12}{84} = \frac{23}{84}$$