Question 1 Pick 1 questions, 0 pts per question

Question

1. Given the differential equation $y' = \sqrt{1 - y^2}$. Use the existence of a unique solution theorem to determine if the differential equation has a unique solution at the following given initial conditions.

(3 points) a. $y(1) = 3$

(3 points) b. $y(4) = 1$

Question

1. Given the differential equation $y' = \sqrt{16 - y^2}$. Use the existence of a unique solution theorem to determine if the differential equation has a unique solution at the following given initial conditions.

(3 points) a. $y(1) = 6$

(3 points) b. $y(4) = 1$

Question 2 Pick 1 questions, 0 pts per question

Question

(8 points) 2. Solve the following differential equation: $\frac{dy}{dx} + \frac{2x}{1+x^2} y = xy^2$

Question

(8 points) 2. Solve the following differential equation: $\frac{dy}{dx} - \frac{1}{2x \ln x} y = 2xy^3$

Question 3 Pick 1 questions, 0 pts per question

Question

(8 points) 3. Solve the following differential equation: $y' - y \tan x = 8 \sin^3 x$

Question

(8 points) 3. Solve the following differential equation: $y' + y \cot x = 8 \cos^3 x$

Question 4 Pick 1 questions, 0 pts per question

Question

(8 points) 4. Solve the following differential equation: $\left(y^2\right) dx + \left(x^2 + xy\right) dy = 0$

Question 5 Pick 1 questions, 0 pts per question

Question

(6 points) 5. Solve the following differential equation: $\frac{dy}{dx} = \frac{x^2(y^2+1)}{x^3+3}$

Question

(6 points) 5. Solve the following differential equation: $\frac{dy}{dx} = \frac{x(y^2+1)}{x^2+3}$

Question 6 Pick 1 questions, 0 pts per question

Question

(8 points) 6. Solve the following differential equation:

$$\left[y^2 \cos x - 3x^2y - 2x\right] dx + \left[2y \sin x - x^3 + \ln y\right] dy = 0$$

Question 7 Pick 1 questions, 0 pts per question

Question

(8 points) 7. Solve the following initial-value problem: $x\frac{dy}{dx} + y = 6x + 1, y(1) = 7$

Question

(8 points) 7. Solve the following initial-value problem: $x\frac{dy}{dx} + y = 2x + 1, y(1) = 6$

Question 8 Pick 1 questions, 0 pts per question

Question

(5 points) 8. Use Euler's method with the following function and given step size to find y_1, y_2, y_3. Use at least four decimal places for your answers.

$$y' = xy^2 + y, \ y(2) = 1, \ h = 0.1$$

Question

(5 points) 8. Use Euler's method with the following function and given step size to find y_1, y_2, y_3. Use at least four decimal places for your answers.

$$y' = xy^2 + x, \ y(1) = 2, \ h = 0.1$$

Question 9 Pick 1 questions, 0 pts per question

Question

9. Given the following differential equation: $y' = 24 - 2y - y^2$

(2 points) a. Determine all equilibrium solutions.

(3 points) b. Determine the intervals along the y-axis on the phase portrait where the solution curves are increasing and determine the intervals where the solution curves are decreasing.

(3 points) c. Determine the intervals along the y-axis on the phase portrait where the solution curves are concave up and the intervals where the solution curves are concave down.

(3 points) d. Sketch representative solution curves in each region determined by parts a, b, and c.

(2 points) e. Classify each equilibrium solution as stable, unstable, or semi-stable.

Question

9. Given the following differential equation: $y' = 15 - 2y - y^2$

(2 points) a. Determine all equilibrium solutions.

(3 points) b. Determine the intervals along the y-axis on the phase portrait where the solution curves are increasing and determine the intervals where the solution curves are decreasing.

(3 points) c. Determine the intervals along the y-axis on the phase portrait where the solution curves are concave up and the intervals where the solution curves are concave down.

(3 points) d. Sketch representative solution curves in each region determined by parts a, b, and c.

(2 points) e. Classify each equilibrium solution as stable, unstable, or semi-stable.

Question 10 Pick 1 questions, 0 pts per question

Question

(4 points) 10. Determine the integrating factor necessary to convert the following differential equation into an exact equation. Verify that the differential equation that results after applying the integrating factor is exact. You do not need to solve the resulting differential equation.

$$\left(x^2 y\right) dx + y\left(x^3 + e^{-3y} \sin y\right) dy = 0$$

Question 11 Pick 1 questions, 0 pts per question

Question

(8 points) 11. A tank initially contains 60 L of a solution in which 10 g of salt is dissolved. A solution with a salt concentration of 3 g/L is added at a rate of 4 L/min. The solution is kept well mixed and is drained from the tank at a rate of 2 L/min. Find the concentration of the solution in the tank after 30 minutes.

Question

(8 points) 11. A tank initially contains 80 L of a solution in which 20 g of salt is dissolved. A solution with a salt concentration of 4 g/L is added at a rate of 4 L/min. The solution is kept well mixed and is drained from the tank at a rate of 2 L/min. Find the concentration of the solution in the tank after 30 minutes.

Question 12 Pick 1 questions, 0 pts per question

Question

12. An object whose initial temperature is $60° F$ is place in a hot oven whose temperature is $350° F$. After 20 minutes, the object's temperature is $150° F$.

(5 points) a. Find the temperature of the object after 30 minutes.

(3 points) b. Find the time when the object reaches a temperature of $250° F$.

Question

12. An object whose initial temperature is $50° F$ is place in a hot oven whose temperature is $300° F$. After 15 minutes, the object's temperature is $120° F$.

(5 points) a. Find the temperature of the object after 30 minutes.

(3 points) b. Find the time when the object reaches a temperature of $200° F$.