MATH 290 – QUIZ #1 *Name:* Key

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(5 points) 1. Solve the following differential equation: $\frac{dy}{dx} = \frac{y+2}{x \ln x}$

$$\frac{dy}{y+2} = \frac{dx}{x \ln x}$$

$$\int \frac{dy}{y+2} = \int \frac{dx}{x \ln x}$$

$$\ln|y+2| = \ln|\ln x| + C$$

$$y+2 = C \ln x$$

(2 points) 2. Given that $y = c_1 e^{2x} + c_2 e^{-x}$ is a solution to a first-order differential equation. Find a solution to the corresponding initial-value problem given the initial condition of $y(0) = 3$, $y'(0) = -2$.

$$y = c_1 e^{2x} + c_2 e^{-x}$$

$$y' = 2c_1 e^{2x} - c_2 e^{-x}$$

$$3 = c_1 + c_2$$

$$-2 = 2c_1 - c_2$$

$$1 = 3c_1$$

$$c_1 = 1/3$$

$$3 = 1/3 + c_2$$

$$c_2 = 8/3$$

$$y = \frac{1}{3} e^{2x} + \frac{8}{3} e^{-x}$$

3. Given the differential equation $\frac{dy}{dx} = y^2 - 8y + 12$.

(1 point) *a.* Determine all equilibrium solutions.

$$\frac{dy}{dx} = (y-6)(y-2)$$

$$(y-6)(y-2) = 0$$

$$y = 2, 6$$

(3 points) *b.* Determine the regions when the solutions are increasing or decreasing.

		2		6	
$y-6$	$-$		$-$		$+$
$y-2$	$-$		$+$		$+$
	$+$		$-$		$+$

increasing on $(-\infty, 2) \cup (6, \infty)$

decreasing on $(2, 6)$

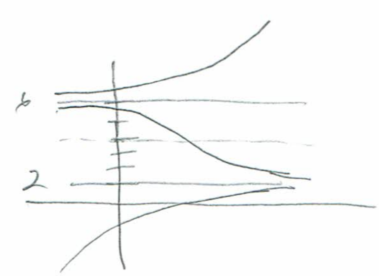
(3 points) *c.* Determine the regions when the solutions are concave up or concave down.

$$\frac{d^2y}{dx^2} = (2y-8)\frac{dy}{dx} = 2(y-4)(y-6)(y-2)$$

		2		4		6	
$2(y-4)$	$-$		$-$		$+$		$+$
$(y-6)$	$-$		$-$		$-$		$+$
$(y-2)$	$-$		$+$		$+$		$+$
	$-$		$+$		$-$		$+$

concave up on $(2, 4) \cup (6, \infty)$

concave down on $(-\infty, 2) \cup (4, 6)$



(2 points) *d.* Classify the equilibrium solutions as stable or unstable.

$y = 2$ stable

$y = 6$ unstable