MATH 290 – QUIZ #1 *Name:* KEY

Directions: Please show all work for maximum credit. This quiz is worth 16 points. Good luck!

(5 points) 1. Solve the following differential equation: $\frac{dy}{dx} = \frac{2x(y-1)}{x^2+3}$

$$\frac{dy}{y-1} = \frac{2x}{x^2+3}\,dx$$

$$\int \frac{1}{y-1}\,dy = \int \frac{2x}{x^2+3}\,dx$$

$$\ln|y-1| = \ln|x^2+3| + C$$

(2 points) 2. Given that $y = x^2 + 2x + c$ is a solution to a first-order differential equation. Find a solution to the corresponding initial-value problem given the initial condition of $y(2) = 1$.

$$1 = 2^2 + 2(2) + C$$

$$1 = 4 + 4 + C$$

$$-7 = C$$

$$y = x^2 + 2x - 7$$

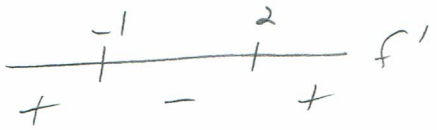
2. Given the differential equation $\frac{dy}{dx} = y^2 - y - 2$.

(1 point) *a.* Determine all equilibrium solutions.

$\frac{dy}{dx} = (y-2)(y+1)$

$y = 2, \quad y = -1$

(3 points) *b.* Determine the regions when the solutions are increasing or decreasing.

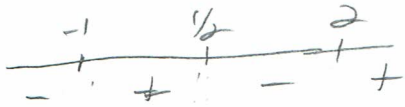


increasing on $(-\infty, -1) \cup (2, \infty)$

decreasing on $(-1, 2)$

(3 points) *c.* Determine the regions when the solutions are concave up or concave down.

$\frac{d^2y}{dx^2} = (2y-1)\frac{dy}{dx} = (2y-1)(y-2)(y+1)$



concave down on $(-\infty, -1) \cup (\frac{1}{2}, 2)$

concave up on $(-1, \frac{1}{2}) \cup (2, \infty)$

(2 points) *d.* Classify the equilibrium solutions as stable or unstable.